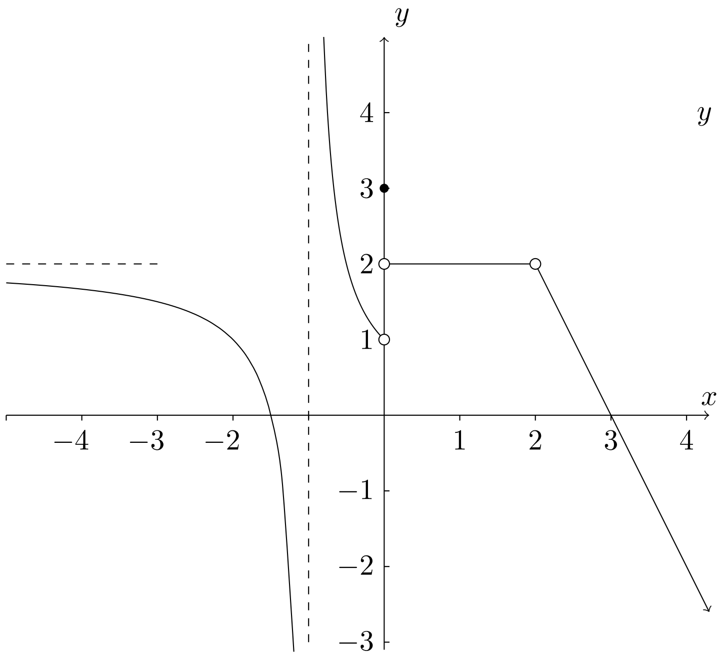
(Marks)

(4) 1. Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



a) $\lim_{x\to 2} f(x) =$ b) $\lim_{x\to -1} f(x) =$ c) $f(0) =$

d) $\lim_{x\to 0^-} f(x) =$ e) $\lim_{x\to -\infty} f(x) =$ f) $f(2) =$

g) List the discontinuities of $f(x)$.

(16) 2. Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" where appropriate.

a) $\displaystyle\lim_{x\to -1} \frac{x^2+3x+2}{2x^2-x-3}$

b) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

c) $\displaystyle\lim_{x\to 3} \frac{\frac{1}{x+1}-\frac{1}{4}}{x-3}$

d) $\displaystyle\lim_{x\to -4} \frac{x+5}{x^2-16}$

e) $\displaystyle\lim_{x\to -\infty} \frac{(x-3)(3x^2+1)}{2x^3-5}$

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f) $\lim_{x\to 5^+} \dfrac{3|5-x|}{x^2-12x+35}$

(3) 3. Find the point(s) of discontinuity for the following function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{7}{x^2+2x-15} & \text{if } x<2 \\ 5 & \text{if } x=2 \\ x^2-5 & \text{if } x>2 \end{cases}$$

(3) 4. Find the value(s) for a and b such that the following function is continuous for all real numbers.

$$f(x) = \begin{cases} 5 & \text{if} \quad x \le 0 \\ ax-b & \text{if} \quad 0<x<8 \\ 3 & \text{if} \quad x \ge 8 \end{cases}$$

(1) 5. **True** or **False**: If $f(c)$ is undefined then $\lim_{x\to c} f(x)$ does not exist. **Briefly** justify your answer.

(1) 6. (a) State the limit definition of the derivative.

(3) (b) Use the definition to find the derivative of $f(x) = \dfrac{1}{3-2x}$.

(20) 7. Find the derivative for each of the following functions. Do not simplify.

a) $y = 8x^3 - \dfrac{4}{x} + \dfrac{5}{\sqrt{(3x)}} + 4\pi^2$

b) $y = 3(2x+5)^2(3x-e^{2x})^6$

c) $y = \log_2(\sqrt{x}) + 4\sqrt[3]{x^5} - 4\cot(2x-3)$

d) $y = \dfrac{\sin^2(4x)}{x^3+2x}$

e) $y = 2x^{x^2+1}$

f) $y = \ln\left[\left(\dfrac{\left(\sqrt{x}-5\right)e^{5x}}{(3x+1)\sec x}\right)^2\right]$

(3) 8. Find the x-coordinates of the point(s) on the curve $y = x^3e^{2x}$ that have horizontal tangents.

(Marks)

(4) 9. Find $\dfrac{d^4y}{dx^4}$ given $y = 3^{2x+1} + \ln(\pi)x^3 - \sin x$.

(5) 10.Let $x^2y^2 + y\ln(x) = 4x$.

a) Find $\dfrac{dy}{dx}$

b) Find the equation of the tangent line to the curve at $(1, 2)$

(10) 11.Given $f(x) = \dfrac{2x^2 - 18}{x^2 - 4}$ with $f'(x) = \dfrac{20x}{(x^2-4)^2}$ and $f''(x) = \dfrac{-20(3x^2+4)}{(x^2-4)^3}$,

(a)Find the x- and y- intercepts (if any).

(b)Find the vertical and horizontal asymptotes (if any).

(c)Give the intervals where $f(x)$ is increasing and decreasing, and the relative extrema (if any).

(d)Give the intervals where $f(x)$ is concave up and concave down, and the points of inflection (if any).

(e)Sketch a labelled graph of $f(x)$.

(3) 12.Use the second derivative test to find all the relative (local) extrema of $f(x) = (x^2 - 9)^2$

(3) 13.Find the absolute (global) extrema of $f(x) = x^3(x-5)^2$ on the interval $[1, 4]$.

(5) 14.A rectangular storage container with an open top is to have a volume of 32 m^3. The length of the base is four times its width. Material for the base costs \$5 per square meter. Material for the sides costs \$4 per square meter. Find the cost of materials for the cheapest such container.

(5) 15.Let the revenue function of a product be given by $R = -0.003x^2 + 5x$ and the average cost function be given by $\bar{C}(x) = \dfrac{300}{x} + 1.1$, where $0 \le x \le 800$.
a) Determine the production level that will maximize profit.
b) At this production level, what comparison can we make regarding marginal revenue and marginal cost?

(5) 16.A sugar shack has been trying to attract more customers and introduced the following pricing scheme: Groups up to ten pay \$20 per ticket. For each additional ticket, all group members receive a \$0.20 discount. What size group would maximize the revenue of this sugar shack?

(6) 17.The demand curve for a product is given by $x = 3000 - 250p^2$ where x is the production level and p is the unit price in dollars.

(Marks)

a) Determine the price elasticity of demand function η.
b) What is the price elasticity of demand when the price is \$3? Is demand elastic, inelastic or unit elastic?
c) At the price of \$3, if the price decreases by 3%, how is demand affected?
d) Determine the price that would maximize the revenue.

ANSWERS

1.a) 2 b) DNE c) 3 d) 1 e) 2 f) DNE g) Discontinuities at $x = -1, x = 0, x = 2$

2.a) $-\frac{1}{5}$ b) $\frac{1}{6}$ c) $-\frac{1}{16}$ d) DNE e) $\frac{3}{2}$ f) $-\frac{3}{2}$

3) Discontinuities at $x = -5, x = 2$ 4) $a = 1, b = -5$ 5) False

6) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ b) $\frac{2}{(3-2x)^2}$

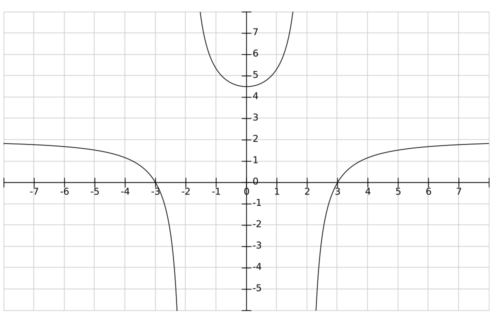
7.a) $y' = 24x^2 + \frac{4}{x^2} - \frac{5}{2\sqrt{3}}x^{-3/2}$ b) $y' = 3(2)(2x+5)(2)(3x-e^{2x})^6 + 3(2x+5)^2(6)(3x-e^{2x})^5(3-2e^2x)$

c) $y' = \frac{1}{2x\ln 2} + 4\frac{3}{5}x^{2/3} - 4\csc^2(2x-3)$ d) $y' = \frac{2\sin(4x)\cos(4x)(4)(x^3+2x)\sin^2(4x)(3x^2+2)}{(x^3+2x)^2}$

e) $y' = 2x^{x^2+1}\left(2x\ln x + \frac{x^2+1}{x}\right)$ f) $y' = 2\left(\frac{1}{2(x-5)} + 5 - \frac{3}{3x+1} - \tan x\right)$

8) $x = 0, \frac{-2}{3}$ 9) $y' = 3^{2x+1}(2\ln 3)^4 - \sin x$ 10) $y' = \frac{-(2x^2y^2 - 4x + y)}{(2x^3y + x\ln(x))}$

11) Intercepts $(3,0), (-3,0), (0,4.5)$ b) VA:$x = -2, x = 2$ HA: $y = 2$
c) INC:$(0,2) \cup (2,\infty)$ DEC:$(-\infty,-3) \cup (-3,0)$ Local Max: NONE, local Min: $(0,4.5)$
d) CU:$(-2,2)$ CD:$(-\infty,2) \cup (2.\infty)$, no inflection points.



12) Local Max: $(0,81)$, Local Mins: $(-3,0), (3,0)$
13) Abs Max of 108 at $x = 3$, Abs Min of 16 at $x = 1$ 14) 2m × 8m × 2m
15.a) A production level of 650. b) They are equal ($R' = C'$) 16) A group of 45 people

17.a) Taking η as a function of p, $\eta = \frac{p}{xp'} = \frac{-500p^2}{3000 - 250p^2}$ b) $\eta(3) = -6$. Elastic

c) Demand increases by 18% d) At a price of \$2