Calculus II Social Science
Final Examination - Winter 2017

#1 Evaluate the following integrals.

(a) $\displaystyle\int \frac{2x^{5/2} - \pi\sqrt{x} + 5x^{-1/2} + \sqrt{x}\sec^2 x}{\sqrt{x}}\, dx$

(b) $\displaystyle\int_0^5 |4x - 12|\, dx$ (c) $\displaystyle\int \frac{\ln(3x+2)}{3x+2}\, dx$

(d) $\displaystyle\int \frac{x^4 - 5x^3 + 4x^2 + 34x - 16}{x^3 - 2x^2 - 8x}\, dx$

(e) $\displaystyle\int \frac{\tan(4 + 5\sqrt{x})}{\sqrt{x}}\, dx$ (f) $\displaystyle\int 2x\sec(5x)\tan(5x)\, dx$

(g) $\displaystyle\int_{\pi/6}^{\pi/3} \sin(3x)\sqrt{\cos(3x) + 4}\, dx$

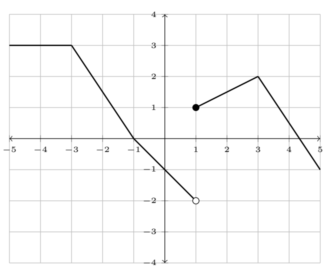
(h) $\displaystyle\int_{-1}^{3} f(x)\, dx$, where $f(x) = \begin{cases} 4x - 2 & \text{if } x < 1 \\ e^x + 3 & \text{if } x > 1 \end{cases}$

#2 The marginal cost of producing a good is given by the equation $\dfrac{dC}{dx} = \dfrac{12}{\sqrt[3]{12x+1}}$. The fixed costs for producing the good are 100 (that is, $C = 100$ when $x = 0$). Find the cost function.

#3 Use the trapezoidal rule with $n = 6$ to estimate $\displaystyle\int_{-1}^{2} e^{x^2-1}\, dx$. Your answer should be correct to three decimal places.

#4 Find the area of the region completely enclosed by the functions $f(x) = x^3 + x^2 - 5x$ and $g(x) = x^2 - x$.

#5 Use the graph of f below to evaluate the definite integrals:



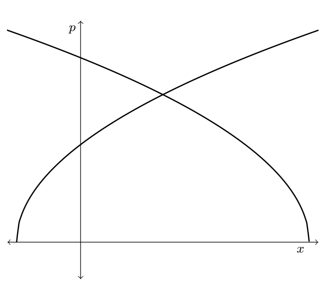
(a) $\displaystyle\int_{-4}^{0} f(x)\, dx$ (b) $\displaystyle\int_{3}^{5} f(x)\, dx$ (c) $\displaystyle\int_{-1}^{3} f(x)\, dx$

(d) $\displaystyle\int_{2}^{2} f(x)\, dx$ (e) $\displaystyle\int_{1}^{-1} f(x)\, dx$

#6 Given the demand function $p = \sqrt{25 - 0.1x}$ and the supply function $p = \sqrt{7 + 0.1x}$:

(a) Find the equilibrium point.

(b) A graph of the demand and supply functions is given below. Label each function, the equilibrium price, the equilibrium point, and the regions whose areas represent the consumer and producer surpluses.



(c) Find the producer surplus.

#7 Find the function y that satisfies the separable differential equation $(x^2 - e^2)y' = \dfrac{x}{y^2}$ and passes through the point $(0, 3)$.

#8 The number of bacteria inside some given dairy product grows at a rate proportional to the number of bacteria present. Suppose that when the product is opened there are 2,000 bacteria present and that after 3 days the number of bacteria has doubled.

(a) Write the differential equation for the situation.

(b) Solve the differential equation.

(c) The product should not be eaten after the bacteria count reaches 5,000,000. How long after opening the product will it take for it to become unsafe?

#9 Evaluate the following limits, using l'Hospital's rule where appropriate.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln(2x+1)}{\sqrt{2x+1}}$ (b) $\displaystyle\lim_{x\to 0} \frac{x\sin(4x)}{x^2 - e^x}$ (c) $\displaystyle\lim_{x\to 0} \frac{x^2}{1 - \cos x}$

#10 Evaluate the following improper integrals:

(a) $\displaystyle\int_4^{\infty} \frac{2}{x\sqrt[3]{\ln x}}\, dx$ (b) $\displaystyle\int_1^2 \frac{x+2}{\sqrt{x^2 + 4x - 5}}\, dx$

#11 Find a formula for the n^{th} term of the sequence given by the formula $\{a_n\} = \left\{-\frac{8}{5}, \frac{11}{15}, -\frac{14}{45}, \frac{17}{135}, \dots\right\}$.

#12 Determine whether the following sequences converge or diverge. If a sequence converges, find its limit. If a sequence diverges, explain why.

(a) $a_n = \dfrac{5n! + 7}{(n+1)!}$ (b) $a_n = \dfrac{(-1)^n 3^n}{3^n + 2}$

#13 The n^{th} partial sum of a sequence $\{a_n\}$ is given by $s_n = \dfrac{3n+2}{2n-1}$. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$.

#14 Determine whether the following series converge or diverge. Justify your answers by referencing a convergence test, and showing why that test may be applied. **In the case of a convergent geometric or telescoping series, find the sum of the series.**

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^{n+1}}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \left[e^{\frac{1}{n}} - e^{\frac{1}{n+2}}\right]$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{2^{n+1}(n-1)!}$ (e) $\displaystyle\sum_{k=1}^{\infty} 2k^{-\sqrt{2}}$

#15 The initial balance of a savings account is \$0. Every two months, \$200 is deposited into the account. Find the account's balance after 5 years if it earns 3% annual interest, compounded every two months.

Answers:

#1 (a) $\frac{2}{3}x^3 - \pi x + 5\ln|x| + \tan x + C$ (b) 26

(c) $\dfrac{\ln^2(3x+2)}{6} + C$ (d) $\frac{1}{2}x^2 - 3x + \ln\left|\dfrac{x^2(x-4)^5}{x+2}\right| + C$

(e) $\frac{2}{5}\ln|\sec(4 + 5\sqrt{x})| + C$

(f) $\frac{2}{5}x\sec(5x) - \frac{2}{25}\ln|(\sec(5x) + \tan(5x)| + C$

(g) $\frac{2}{9}[8 - 3^{3/2}]$ (h) $2 - e + e^3$

#2 $C(x) = \frac{3}{2}(12x+1)^{2/3} + \frac{197}{2}$

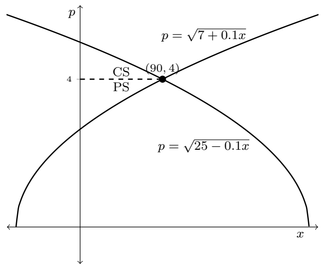
#3 ~ 8.173

#4 8

#5 (a) 11/2 (b) 1 (c) 1 (d) 0 (e) 2

#6 (a) $(90, 4)$

(b)



(c) 56.8017

#7 $y = (\frac{3}{2}\ln|x^2 - e^2| + 24)^{1/3}$

#8 (a) $\dfrac{dN}{dt} = kN$ (b) $N = 2000 \cdot e^{\frac{\ln 2}{3}t}$ (c) ~ 33.86 days.

#9 (a) 0 (b) 0 (c) 2

#10 (a) diverges (b) $\sqrt{7}$

#11 $a_n = \dfrac{(-1)^n(3n+5)}{5 \cdot 3^{n-1}}$

#12 (a) converges to 0 (b) diverges due to oscillation

#13 3/2

#14 (a) $-\frac{8}{15}$ (GST) (b) Div. (RT) (c) $e + \sqrt{e} - 2$ (TS)

(d) Conv. (RT) (e) Conv. (PS)

#15 \$6488.28