1. Evaluate the following limits. Write $\infty,-\infty$ of $D N E$, as approriate.
(a) (3 points) $\lim _{x \rightarrow-4^{+}} \frac{x+\sqrt{20+x}}{x+4}$
(b) (3 points) $\lim _{x \rightarrow 3^{-}} \frac{9-x^{2}}{x^{2}-6 x+9}$
(c) (3 points) $\lim _{x \rightarrow-2^{-}} \frac{x^{2}-3 x-10}{|2 x+4|}$
(d) $\left(3\right.$ points) $\lim _{x \rightarrow 0} \frac{\tan (8 x)+\sin (4 x)}{\sin (3 x)}$
2. (5 points) Find the vertical and horizontal asymptotes of $f(x)=\frac{7 x-\sqrt{4 x^{2}+3 x+2}}{x+8}$.
3. (4 points) What value(s) of $k$ make $g(x)$ continuous on $\mathbb{R}$ ?

$$
g(x)=\left\{\begin{array}{lll}
x^{2}+4-3 k & , \text { if } x<0 \\
2 x+k^{2} & , \text { if } 0 \leq x \leq 2 \\
x^{2} k+x-k & , \text { if } x>2
\end{array}\right.
$$

4. (4 points) Given $f(x)=\frac{2}{3 x-4}$, find $f^{\prime}(x)$ by using the definition of the derivative.
5. (4 points) Find an equation for the tangent line to $f(x)=3 \tan x+\cos (2 x)$ at $x=\pi / 4$.
6. Find $\frac{d y}{d x}$ in each case. Do not simplify your answer.
(a) (4 points) $y=5 x^{6}-\frac{5}{2 x^{6}}-2 \sqrt[6]{x^{5}}+7 \cdot 5^{x}+\sin \left(\frac{\pi}{2}\right)$
(b) (4 points) $y=\frac{x^{3} \cos x}{e^{4 x}-(2 x+3)^{2}}$
(c) (4 points) $y=\tan ^{3}(\csc (\sqrt{4 x})+x)$
(d) (4 points) $y=\sec x+(\ln x)^{4 x}$
7. (4 points) Find the 34 th derivative of $f(x)=\sin (3 x)-e^{-x}+x^{28}$
8. (5 points) Find the equation for the line normal to the curve $\frac{y}{x}+x y=-2$ at the point $(1,-1)$.
9. (10 points) Consider the following function, along with its first and second derivatives.
$f(x)=\frac{x^{2}}{(x+2)^{2}}, f^{\prime}(x)=\frac{4 x}{(x+2)^{3}}, f^{\prime \prime}(x)=\frac{8-8 x}{(x+2)^{4}}$
(a) Find the domain and intercepts of $f$.
(b) Find the vertical and horizontal asymptotes of $f$ (if any).
(c) Find the intervals of increase/decrease of $f$.
(d) Find the local (relative) extrema of $f$.
(e) Find the intervals of concavity of $f$.
(f) Find all points of inflection of $f$.
(g) On the next page, sketch the graph of $f$.
10. (4 points) Find the absolute extrema of $f(x)=(x-6) \cdot \sqrt[3]{x-2}$ on the interval $[1,10]$.
11. (6 points) A 13-foot ladder is leaning against a house. It's bottom starts to slide away. By the time the bottom of the ladder is 12 feet from the house, it's moving at the rate $6 \mathrm{ft} / \mathrm{sec}$. At what rate is the angle $\theta$ between the ladder and the wall changing at that moment?
12. (6 points) Find the dimensions (length and width) of the rectangle of largest area that can be inscribed in an equilateral triangle with sides of length 6 cm if one side of the rectangle lies on the base of the triangle.
13. Evaluate the following integrals.
(a) $(3$ points $) \int_{1}^{2}\left(3 e^{x}-\frac{3}{x}+\sqrt{3}\right) d x$
(b) (3 points) $\int\left(\frac{2-\sin ^{2} x}{\cos ^{2} x}\right) d x$
14. (3 points) Find $f$ if $f^{\prime}(x)=x-\frac{1}{\sqrt{x}}$, and $f(4)=0$.
15. (3 points) Express the integral $\int_{-\pi / 6}^{\pi / 2} \cos ^{2} x d x$ as a limit of Riemann sums. Do not evaluate the limit or the integral.
16. (5 points) Consider the function

$$
f(x)= \begin{cases}x+4 & \text { if } x \leq 0 \\ \sqrt{16-x^{2}} & \text { if } 0<x \leq 4\end{cases}
$$

(a) Sketch the function $f$ from $x=-6$ to $x=4$.
(b) Evaluate $\int_{-6}^{4} f(x) d x$ by interpreting the integral as areas. Do not use antiderivatives.
17. (3 points) Find the derivative of the function $f(x)=\int_{x^{2}}^{16} \sqrt{1+t^{2}} d t$

## Answers

1. 

(a) $9 / 8$
(b) $\infty$
(c) $7 / 2$
(d) 4
2. Vertical asymptote : $x=-8$, Horizontal asymptote : $y=5$ and $y=9$.
3. $k=1$
4. $f^{\prime}(x)=\frac{-6}{(3 x-4)^{2}}$
5. $y-3=4(x-\pi / 4)$
6.
(a) $y^{\prime}=30 x^{5}+15 x^{-7}-\frac{5}{3} x^{-1 / 6}+7 \cdot 5^{x} \ln 5$
(b) $y^{\prime}=\frac{\left(3 x^{2} \cos x-x^{3} \sin x\right)\left(e^{4 x}-(2 x+3)^{2}\right)-x^{3} \cos x\left(4 e^{4 x}-4(2 x+3)\right)}{\left(e^{4 x}-(2 x+3)^{2}\right)^{2}}$
(c) $y^{\prime}=3 \tan ^{2}(\csc (2 \sqrt{x})+x) \sec ^{2}(\csc (2 \sqrt{x})+x)\left(-\frac{\csc (2 \sqrt{x}) \cot (2 \sqrt{x})}{\sqrt{x}}+1\right)$
(d) $y^{\prime} \sec x \tan x+(\ln x)^{4 x}\left(4 \ln (\ln x)+\frac{4}{\ln x}\right)$
7. $f^{(34)}(x)=-\sin (3 x) 3^{34}-e^{-x}$
8. $x=1$
9.
(a) $D: \mathbb{R} \backslash\{-2\}$, the only intercept is $(0,0)$.
(b) Vertical asymptote : $x=-2$, Horizontal asymptote : $y=1$.
(c) $f$ increases on $(-\infty,-2) \cup(0, \infty) . f$ decreases on $(-2,0)$.
(d) $f$ has a relative minimum at $(0,0)$.
(e) $f$ is concave up on $(-\infty,-2) \cup(-2,1) . f$ is concave down on $(1, \infty)$.
(f) $(1,1 / 9)$ is a point of inflection of $f$.

(g)
10. Absolute max : $(10,8)$, absolute min : $(3,-3)$
11. The angle is changing at a rate of $6 / 5 \mathrm{rad} / \mathrm{s}$
12. The rectangle of largest area has a length of 3 cm and a width of $\frac{3 \sqrt{3}}{2} \mathrm{~cm}$.
13.
(a) $3 e^{2}-3 e-3 \ln (2)+\sqrt{3}$
(b) $\tan x+x+C$
14. $f(x)=\frac{x^{2}}{2}-2 \sqrt{x}-4$
15. $\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \cos ^{2}\left(-\frac{\pi}{6}+\frac{2 \pi}{3 n} i\right) \frac{2 \pi}{3 n}$
16.
(a)

(b) $6+4 \pi$
17. $f^{\prime}(x)=-2 x \sqrt{1+x^{4}}$

