Advanced problems

Factoring (with sums and differences of cubes)

Factor completely:

1. $x^3 - 8$
2. $27 - y^3$
3. $z^3 + 125$
4. $x^3 + 64$
5. $8t^3 + 1$
6. $8 - 27y^3$
7. $64z^3 - 27$
8. $125x^3 + 8$
9. $17 - 17x^3$
10. $x^4 + x$
11. $8xy - xy^4$
12. $54a^5b^2 - 2a^2b^2$
13. $32x^5 + 4x^2$
14. $16d^4 - 2d$
15. $27x^5 - x^2$
16. $8x^7 - 64x^4$
17. $24x^2 + 3x^5$
18. $16s^4 - 2st^3$

Logarithmic Equations

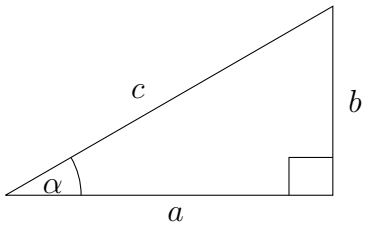
Solve for x.

19. $\log_2 x = 3$
20. $\log_3 x = 2$
21. $\log_5(2x + 3) = 1$
22. $\log_2(2^x) = 971$
23. $5 \log_{10}(x + 2) = 15$
24. $\frac{1}{6} \log_5 x = \frac{1}{2}$
25. $\log_{10}(x + 2) = \log_{10}(2x - 4)$
26. $\ln(11x) = \ln(3 - x)$
27. $\log_2(x^2) = \log_2(16)$

Trigonometry

28. Suppose $\sin\alpha = 3/5$. Find the value of $\cos\alpha$ and the value of the expression $\sin^2\alpha + \cos^2\alpha$.

29. Referring to the picture below, how much is $\sin^2\alpha + \cos^2\alpha$? Completely simplify your answer. (Hint: $c^2 = a^2 + b^2$).



30. Referring to the picture in the previous exercise, how much is $\sec^2\alpha - \tan^2\alpha$?

Evaluate:

31. $\sin^2 45^\circ + \sin 30^\circ - \tan 45^\circ$;
32. $\sin^2 30^\circ + \cos^2 30^\circ$
33. $\csc^2 30^\circ - \cot^2 30^\circ$;
34. $\left(\tan^2 60^\circ - \cot 45^\circ\right)^2 - \sec^2 45^\circ$;

Answers:

1. $(x-2)(x^2+2x+4)$
2. $(3-y)(9+3y+y^2)$
3. $(z+5)(z^2-5z+25)$
4. $(x+4)(x^2-4x+16)$
5. $(2t+1)(4t^2-2t+1)$
6. $(2-3y)(4+6y+9y^2)$
7. $(4z-3)(16z^2+12z+9)$
8. $(5x+2)(25x^2-10x+4)$
9. $17(1-x)(1+x+x^2)$
10. $x(x+1)(x^2-x+1)$
11. $xy(2-y)(4+2y+y^2)$
12. $2a^2b^2(3a-1)(9a^2+3a+1)$
13. $4x^2(2x+1)(4x^2-2x+1)$
14. $2d(2d-1)(4d^2+2d+1)$
15. $x^2(3x-1)(9x^2+3x+1)$
16. $8x^4(x-2)(x^2+2x+4)$
17. $3x^2(x+2)(x^2-2x+4)$
18. $2s(2s-t)(4s^2+2st+t^2)$
19. 8
20. 9
21. 1
22. 971
23. 998
24. 125
25. 6
26. $\frac{1}{4}$
27. ± 4
28. $\cos\alpha = 4/5$; $\sin^2\alpha + \cos^2\alpha = 1$
29. $\sin^2\alpha + \cos^2\alpha = 1$
30. $\sec^2\alpha - \tan^2\alpha = 1$
31. 0
32. 1
33. 1
34. 2