Solutions to Exponential Functions Problem Set

1. Find the value of:

(a) e^0
(b) $8^{-1/3}$
(c) 2^{-5}
(d) $\left(\frac{1}{4}\right)^{-3/2}$
(e) $27^{-2/3}$
(f) $2 \cdot 3^{5-4x}$, when $x = 2$

a) $e^0 = \boxed{1}$
(a^0 is always 1)

b) $8^{-1/3} = \frac{1}{8^{1/3}}$
$= \frac{1}{\sqrt[3]{8}} = \boxed{\frac{1}{2}}$

c) $2^{-5} = \frac{1}{2^5} = \boxed{\frac{1}{32}}$

d) $\left(\frac{1}{4}\right)^{-3/2} = \left(\left(\frac{1}{4}\right)^{1/2}\right)^{-3}$
$= \left(\frac{1}{\sqrt{4}}\right)^{-3}$
$= \left(\frac{1}{2}\right)^{-3}$
$= 2^3$
$= \boxed{8}$

e) $27^{-2/3} = \left((27)^{1/3}\right)^{-2}$
$= \left(\sqrt[3]{27}\right)^{-2}$
$= 3^{-2}$
$= \frac{1}{3^2} = \boxed{\frac{1}{9}}$

f) $2 \cdot 3^{5-4(2)}$
$= 2 \cdot 3^{5-8} = 2 \cdot 3^{-3} = \frac{2}{3^3} = \boxed{\frac{2}{27}}$

2. Solve for x:

(a) $2^x = 0$
(b) $e^x = -3$
(c) $3^x = \frac{1}{9}$
(d) $9^x = 27$
(e) $e^x = 1$
(f) $4 - 3^{x^2-1} = 1$
(g) $6^{2x-1} = 36$
(h) $e^{4x^2-8} = e$
(i) $4^{x-1} = 8^x$
(j) $5^x = 25^{3x-1}$

a) No solution, 2^x will always give a positive result.

b) No solution, e^x will always give a positive result.

c) $3^x = \frac{1}{9}$
$3^x = \frac{1}{3^2}$
$3^x = 3^{-2}$
so $\boxed{x = -2}$

d) $9^x = 27$ Reduce to a common base
$(3^2)^x = 3^3$
$3^{2x} = 3^3$
$2x = 3$
$\boxed{x = \frac{3}{2}}$

e) $e^0 = 1$, so $\boxed{x = 0}$

f) $4 - 3^{x^2-1} = 1$
$4 - 1 = 3^{x^2-1}$
$3 = 3^{x^2-1}$
$3^1 = 3^{x^2-1}$
$1 = x^2 - 1$
$x^2 = 2$
$\boxed{x = \pm\sqrt{2}}$

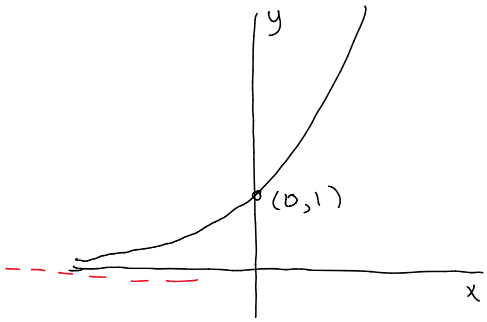
g) $6^{2x-1} = 6^2$
$2x - 1 = 2$
$2x = 3$
$\boxed{x = \frac{3}{2}}$

h) $e^{4x^2-8} = e^1$
$4x^2 - 8 = 1$
$4x^2 - 9 = 0$
$(2x-3)(2x+3) = 0$
$2x - 3 = 0$ or $2x + 3 = 0$
$\boxed{x = \frac{3}{2}}$ $\boxed{x = -\frac{3}{2}}$

i) $4^{x-1} = 8^x$ reduce to common base
$(2^2)^{x-1} = 2^3$
$2^{2x-2} = 2^3$
$2x - 2 = 3$
$2x = 5$
$\boxed{x = \frac{5}{2}}$

j) $5^x = 25^{3x-1}$
$5^x = (5^2)^{3x-1}$
$5^x = 5^{6x-2}$
$x = 6x - 2$
$2 = 5x$
$\boxed{x = \frac{5}{2}}$

3. Draw a sketch of the function $y = a^x$, if $a > 1$. Make a list of features that all graphs of this type have in common.



- When $x=0$, $y=1$ (because $a^0=1$)
- When x is a large negative number, y is a small positive number (horizontal asymptote at $y=0$ on the left)
- When x is a large positive number, y is an even bigger positive number (y grows fast on the right side of the graph)
- y is always positive

4. Simplify the expression $\dfrac{2e^{\frac{5x}{3}} + e^{\frac{2x}{3}}}{2e^{2x} + e^{x}}$, by first factoring out the greatest common factor from the numerator and from the denominator.

$$\frac{2e^{\frac{5x}{3}} + e^{\frac{2x}{3}}}{2e^{2x}+e^{x}} = \frac{e^{2x/3}\left(2e^{\frac{3x}{3}}+1\right)}{e^{x}\left(2e^{x}+1\right)} = \frac{e^{2x/3}(2e^{x}+1)}{e^{x}(2e^{x}+1)} = \frac{e^{2x/3}}{e^{x}} = \boxed{\frac{1}{e^{x/3}}}$$

$=1$

5. Use properties of exponents to simplify the expression $\dfrac{4e^{x} \cdot 4^{7x}}{e^{3-x}}$

$$\frac{4e^{x}\cdot 4^{7x}}{e^{3-x}} = \frac{e^{x}}{e^{3-x}}\cdot\left(4\cdot 4^{7x}\right) = e^{x-(3-x)}\cdot 4^{1+7x} = \boxed{e^{2x-3}\cdot 4^{7x+1}}$$