Factoring Practice Solutions

Factoring Monic Quadractics Using Product-Sum Method Solutions

1. $x^2 + x - 6 = (x+3)(x-2)$
   This is because $(3)(-2) = -6$ and $3-2 = 1$. We can start by thinking of the fact that $(3)(2) = 6$ and noting that their difference is 1. We know that we are looking for a difference, and not a sum, because the constant term, -6, is negative. We see that the coefficient of x is $+1$, so we know that the difference needs to be $+1$ and not -1. Therefore, we choose the smaller of the values to be negative.

2. $x^2 - 19x + 60 = (x-15)(x-4)$
   This is because $(-15)(-4) = 60$ and $-15-4 = -19$. We know that both values must be negative because the coefficient of $x, -19$, is negative but the constant term, $+60$, is positive. If you have trouble finding the two numbers whose product is 60 and whose sum is -19, I suggest starting with 1 and thinking of all of the possible products that give 60.
   For example, $(1)(60) = 60$, but $1 + 60 = 61$
   $(-2)(-30) = 60$, but $-2-30 = -32$
   $(-3)(-20) = 60$, but $-3-20 = -23$
   $(-4)(-15) = 60$, and $-4-15 = -19$, so we're done!

3. $x^2 + 10x + 25 = (x+5)(x+5) = (x+5)^2$
   This is because $(5)(5) = 25$ and $5+5 = 10$. Because the same number appeared twice, we can just immediately write this as $(x+5)^2$.

4. $x^2 - 3x - 54 = (x+6)(x-9)$
   This is because $(6)(-9) = -54$ and $6-9 = -3$. We know that one of the values must be positive and one must be negative because the constant term, -54, is negative. Because the coefficient of $x, -3$, is negative, we take the larger of the two values to be negative. If you have trouble finding the two values, again I suggest starting with 1.
   $(1)(-54) = -54$, but $1-54 = -53$
   $(2)(-17) = -54$, but $2-17 = -15$
   $(3)(-18) = -54$, but $3-18 = -15$
   54 is not divisible by 4
   54 is not divisible by 5
   $(6)(-9) = -54$, and $6-9 = -3$, so we're done!

5. $x^2 + 5x + 9$ is irreducible.
   We can see this by checking $b^2 - 4ac$. In this case, $a = 1, b = 5$, and $c = 9$, so $b^2 - 4ac = 5^2 - 4(1)(9) = 25 - 36 = -11 < 0$. Since the result is negative, the polynomial is irreducible.

Sum and Difference of Squares Solutions

1. $x^2 - 36 = (x - 6)(x + 6)$
   In this case, we have $x^2 - 6^2$, because $\sqrt{36} = 6$. So our a is x and our b is 6, and we can apply the formula directly.

2. $121 - 4x^2 = (11 - 2x)(11 + 2x)$
   In this case, $\sqrt{121} = 11$ and $\sqrt{4x^2} = 2x^2$. Ot is useful to rewrite as $11^2 - (2x)^2$, so that we can easily see that $a = 11$ and $b = 2x$. It doesn't matter that the x term comes second, we can still just immediately apply the formula.

3. $x^2 + 15$ is irreducible.
   A sum of squares is always irreducible.

4. $x^2 - 15 = (x - \sqrt{15})(x + \sqrt{15})$
   In this case 15 is not a perfect square, but we can still think of this as a difference of squares by rewriting it as $x^2 - (\sqrt{15})^2$.

Sum and Difference of Cubes Solutions

1. $x^3 + 27 = (x + 3)(x^2 - 3x + 9)$
   We can see that $\sqrt[3]{27} = 3$, so we have $x^3 + 3^3$. Thus $a = x$ and $b = 3$ and we can immediately apply the sum of cubes formula.
   The resulting trinomial from the sum or difference of cubes will always be irreducible, so we are done.

2. $8x^3 + 125 = (2x + 5)(4x^2 - 10x + 25)$
   In this case, $\sqrt[3]{8x^3} = 2x$ and $\sqrt[3]{125} = 5$, so $a = 2x$ and $b = 5$, and we directly apply the sum of cubes formula.
   The resulting trinomial from the sum or difference of cubes will always be irreducible, so we are done.

3. $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$
   We can see that $\sqrt[3]{64} = 4$, so we have $x^3 + 3^3$. Thus $a = x$ and $b = 3$ and we can immediately apply the difference of cubes formula.
   The resulting trinomial from the sum or difference of cubes will always be irreducible, so we are done.

4. $216 - 125x^6 = (6 - 5x^2)(36 + 30x^2 + 25x^4) = (\sqrt{6} - \sqrt{5}x)(\sqrt{6} + \sqrt{5}x)(25x^4 + 30x^2 + 36)$
   In this case, $\sqrt[3]{216} = 6$ and $\sqrt[3]{125x^6} = 5x^2$. As such, $a = 6$ and $b = 5x^2$. It doesn't matter that b comes second or that we have x^6 instead of x^3, this can still be thought of as a difference of cubes, and we apply the formula accordingly.
   The trinomial is, of course, irreducible, but the other resulting factor is $6 - 5x^2$. Even though neither 5 nor 6 is a perfect square, we can think of this as $(\sqrt{6})^2 - (\sqrt{5}x)^2$ and see that it is a difference of squares. We thus also apply the difference of squares formula to this part.
   Note: I thought to think of this as a difference of squares because I had a number minus something containing an x^2.

Greatest Common Factor Solutions

1. $4x^4 - 36x^3 = 4x^3(x-9)$
   Here, we have two terms. If we think of 36 as a product of prime factors, we can rewrite this is $4x^4 - (4)(3)(3)x^3$. We can see that in both the first and second terms, there is a 4, so this is part of our GCF. For the xs, we always take the term with the lowest power. So our GCF is $4x^3$. Factoring it out is equivalent to dividing each term by $4x^3$.
   When we remove it from the first term, we only have one x left over. When we remove it from the second term, we have $(3)(3) = 9$ left.

2. $x^4e^x - 16x^3e^x - 36x^2e^x = x^2e^x(x^2 - 16x - 36) = x^2e^x(x+2)(x-18)$
   Here, all 3 terms contain a factor of e^x. The term with the lowest power of x contains x^2. So the GCF is x^2e^x. When we factor it out, the result is $(x^2 - 16x - 36)$, which is a quadratic polynomial that can be factored more. So we must apply the product-sum method. We know that one of the values must be negative and one must be positive because the constant term is negative. We also know that the bigger of the two terms will be negative since the coefficient of $x, -16$, is negative. In this case, $(2)(-18) = -36$ and $2 - 18 = -16$, so we have the two numbers to use for factoring.
   Remember, if you have trouble finding the two numbers, you can always start by thinking of it as a factor of 2 and make a list. In this case, we don't have to go any further.

3. $3x^5y^3 + 5x^2y^2 - 15x^2y^3 = x^2y^2(3x^3y + 5 - 15y)$
   The numbers $3, 5$, and 15 don't all have one common factor. The lowest x power is 2, and the lowest y power is y^2, so the GCF is x^2y^2. In the first term, when we take away 2 of the xs, there are 3 left over, and when we take away 2 of the ys, there is one left over. In the second term, all of the xs and ys are taken away, so all that remains is the constant. And in the third term, all of the xs are taken away but there is one y left over.

4. $17x^5 + 17x^2 = 17x^2(x^3+1) = 17x^2(x+1)(x^2 - x + 1)$
   In this case the GCF is $17x^2$. When we take it from the first term, x^3 is remaining. We take everything away from the second term. As $\frac{17x^2}{17x^2} = 1$, this means a 1 is remaining. When we factor it out, the resulting $(x^3 + 1)$ is a sum of cubes, so we just need to apply the sum of cubes formula to finish factoring.

Factoring By Grouping Solutions

1. $x^3 + 2x^2 + 2x + 4 = (x + 2)(x^2 + 2)$
   This is a cubic polynomial with 4 terms, so we can see that factoring by grouping might be a good idea. First, we separate it into two groups, $(x^3 + 2x^2) + (2x + 4)$ and then factor out the GCF from each of the groups to get $x^2(x + 2) + 2(x + 2)$. This result is now two terms, $x^2(x + 2)$ and $2(x + 2)$. We can see that both terms contain $(x + 2)$, so this is the GCF of the result. We factor out the $x + 2$ to get the final result.
   Note that $x^2 + 2$ is a sum of squares, which can never be factored, so we are done.

2. $2x^3 + 3x^2 - 6x - 12$ In this case, factoring by grouping fails.
   First, we separate into two groups, $(2x^3 + 3x^2) + (-6x - 12)$. Taking the GCF out of each term results in $x^2(2x + 3) - 6(x + 2)$. In this case, the two new terms don't have a GCF, so factoring by grouping fails.

3. $2x^3 + 3x^2 - 18x - 27 = x^2(2x + 3) - 9(2x + 3) = (2x + 3)(x^2 - 9) = (2x + 3)(x - 3)(x + 3)$
   First, we separate into two groups, $(2x^3 + 3x^2) + (-18x - 27)$. Factoring out the GCF of both groups results in $x^2(2x + 3) - 9(2x + 3)$. Since $18 = (2)(3)(3)$ and $27 = (3)(3)(3)$, we can see that the GCF of the second group is $(3)(3) = 9$. Also, since the x value is negative, we factor out the negative. Otherwise, the result would be $x^2(2x + 3) + 9(-2x - 3)$, which would not result in the common factor of $2x + 3$.
   Factoring out the common factor of $2x + 3$, we get $(2x + 3)(x^2 - 9)$. We can recognize that $x^2 - 9$ is a difference of squares, so we apply the difference of squares formula to finish factoring.

4. $30x^3 - 6x^2 + 15x - 3 = (5x - 1)(6x^2 + 3)$
   First, we separate into two groups, $(30x^3 - 6x^2) + (15x - 3)$. Then, we factor out the GCF from each group, resulting in $6x^2(5x - 1) + 3(5x - 1)$. We factor out the new GCF of $5x - 1$ to get $(5x - 1)(6x^2 + 3)$. Lastly, we check if any more factoring can be done. $6x^2 + 3$ is a sum of squares, which can never be factored, so we are done.

Factoring Non-Monic Quadractics Solutions

1. $2x^2 - 7x - 4 = (x - 4)(2x + 1)$
First, $(-4)(2) = -8$, so we want two numbers whose product is -8 and whose sum is -7. Since the sum is negative, we know the bigger of the two numbers must be negative. These numbers are 1 and -8. We use them to separate the $-7x$ into $-8x + x$ so that we can then factor by grouping. $2x^2 - 7x - 4 = 2x^2 - 8x + x - 4 = 2x(x - 4) + (x - 4) = (x - 4)(2x + 1)$.
Note that writing as $2x^2 + x - 8x - 4$ would also work. $2x^2 - 7x - 4 = 2x^2 + x - 8x - 4 = x(2x + 1) - 4(2x + 1) = (2x + 1)(x - 4)$

2. $4x^2 + 17x + 18 = (x + 2)(4x + 9)$
First, $(18)(4) = 72$. So we want two numbers whose product is 72 and whose sum is 17. These two numbers are 8 and 9. We use them to separate the $17x$ into $8x + 9x$ so that we can then factor by grouping. So we have $4x^2 + 8x + 9x + 18 = (4x^2 + 8x) + (9x + 18) = 4x(x + 2) + 9(x + 2) = (x + 2)(4x + 9)$.
Remember that, if you have trouble finding the two numbers, you can start by thinking of 1 as a factor of 72. $(1)(72) = 72$ but $1 + 72 = 73$
$(2)(34) = 72$ but $2 + 34 = 36$
$(3)(24) = 72$ but $3 + 24 = 27$
$(4)(18) = 72$ but $4 + 18 = 22$
72 is not divisible by 5.
$(6)(12) = 72$ but $6 + 12 = 18$
72 is not divisible by 7. $(8)(9) = 72$ and $8 + 9 = 17$, so these are our numbers.
Note that writing as $4x^2 + 9x + 8x + 18$ would also work. $4x^2 + 9x + 8x + 18 = x(4x + 9) + 2(4x + 9) = (4x + 9)(x + 2)$

3. $6x^2 - 23x + 20 = (3x - 4)(2x - 5)$
First, $(6)(20) = 120$, so we want two numbers whose product is 120 and whose sum is -23. Since the product is positive but the sum is negative, this means both of the numbers must be negative. These numbers are -8 and -15. We use them to separate $-23x$ into $-8x - 15x$ so that we can then factor by grouping. $6x^2 - 23x + 20 = 6x^2 - 8x - 15x + 20 = 2x(3x - 4) - 5(3x - 4) = (3x - 4)(2x - 5)$
Remember that, if you have trouble finding the two numbers, you can always start by thinking of 1 as a factor of 120. $(1)(-120) = -120$ but $1 - 120 = -119$
$(-2)(-60) = 120$ but $-2 - 60 = -62$
$(-3)(-40) = 120$ but $-3 - 40 = -43$
$(-4)(-30) = 120$ but $-4 - 30 = -34$
$(-5)(-24) = 120$ but $-5 - 24 = -29$
$(-6)(-20) = 120$ but $-6 - 20 = -26$
120 is not divisible by 7
$(-8)(-15) = 120$, so these are our two numbers.
Note that writing as $6x^2 - 15x - 8x + 20$ would also work. $6x^2 - 15x - 8x + 20 = 3x(2x - 5) - 4(2x - 5) = (3x - 4)(2x - 5)$

Combining it All Solutions

1. $3x^2 + 21x + 30 = 3(x + 5)(x + 2)$
   We could do the product-sum factoring by grouping method for non-monic quadratics here, but factoring out the GCF will make things easier.
   First, we note that this quadratic polynomial has a GCF of 3. Factoring out the 3 results in the monic polynomial $3(x^2 + 7x + 10)$. The two numbers whose product is 10 and whose sum is 7 are 5 and 2. So we get $3(x^2 + 7x + 10) = 3(x + 5)(x + 2)$.

2. $6x^2 - 22x + 20 = 2(x - 2)(3x - 5)$
   We could do the product-sum factoring by grouping method for non-monic quadratics here, but factoring out the GCF will make things easier.
   First, we note that this quadratic polynomial has a GCF of 2. Factoring out 2 results in $2(3x^2 - 11x + 10)$. We still have a non-monic quadratic, but the numbers we're dealing with are much smaller. We see $(10)(3) = 30$, so we want two numbers whose product is 30 and whose sum is -11. The numbers must both be negative so that the sum can be negative while the product remains positive. These two numbers are -5 and -6. So we get $2(3x^2 - 11x + 10) = 2(3x^2 - 6x - 5x + 10) = 2(3x(x - 2) - 5(x - 2)) = 2(x - 2)(3x - 5)$.
   If you need help finding the two numbers,
   $(-1)(-30) = 30$ but $-1 - 31 = -31$
   $(-2)(-15) = 30$ but $-2 - 15 = -17$
   $(-3)(-10) = 30$ but $-3 - 10 = -13$
   30 is not divisible by 4
   $(-5)(-6) = 30$, and $-5 - 6 = -11$, so these are our numbers. Note that we could have also used $-5x - 6x$ instead of $-6x - 5x$, giving $2(3x^2 - 11x + 10) = 2(3x^2 - 5x - 6x + 10) = 4(x(3x - 5) - 2(3x - 5)) = 4(3x - 5)(x - 2)$

3. $216x^3 - 27x^6 = -27x^3(x - 2)(x^2 + 2x + 4)$
   We can see that this is a difference of cubes, so we could directly apply the formula. But, we can also see that there is a GCF. Factoring that out first will make our lives easier. Remember, we always take the smallest power for the GCF! Also note that $216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = (8)(27)$. So the GCF is $27x^3$, resulting in $216x^3 - 27x^6 = -27x^3(-8 + x^3) = -27x^3(x^3 - 8) = -27x^3(x - 2)(x^2 + 2x + 4)$.
   First, I factored out the negative version of the GCF so that the remaining x would be positive. Then I rewrote $-8+x^3$ in the more familiar form of x^3-8. Then I directly applied the difference of cubes. Remember that the resulting trinomial from the sum or difference of cubes will always be irreducible, so we are done.