Solutions to Logarithm Problem Set

1. Convert the exponential equation to a logarithmic equation, or vice versa:

Ex.	$27^{2/3} = 9$	$\log_{27}(9) = \frac{2}{3}$
(a)	$4^{3/2} = 8$	$\log_4(8) = 3/2$ $4^{?} = 8$
(b)	$4^x = \frac{1}{16}$	$\log_4\left(\frac{1}{16}\right) = x$ (By the way, $\frac{1}{16} = 4^{-2}$, so $x = -2$) $4^{?} = \frac{1}{16}$
(c)	$e^3 = y$	$\ln(y) = 3$ ($\log_e$ is typically written as ln or just log)
(d)	$e^1 = e$	$\log_e e = 1$ e^1 $\ln(e) = 1$
(e)	$a^6 = 3$	$\log_a(3) = 6$
(f)	$e^x = 8$	$\ln(8) = x$
(g)	$2^y = -5$ * Note: $\log_2(-5)$ is undefined, y does not exist	$\log_2(-5) = y$

2. Find the value of:

(a) $\log_7\left(\frac{1}{7}\right)$
(b) $\log_{\frac{1}{4}} 2$
(c) $\log_a(a)$
(d) $\ln(0)$
(e) $\ln(e^{x^2-4})$
(f) $5^{2\log_5(3)}$
(g) $\log_4 1$
(h) $\ln(1)$
(i) $\log_4(4)$
(j) $\ln(e)$
(k) $\ln(4e)$
(l) $\ln(e+4)$
(m) $e^{\ln(1)}$
(n) $e^{\ln(-6)}$
(o) $\log_4(-8)$
(p) $\log_2(4^x)$

a) $7^{-1} = \frac{1}{7}$, so $\log_7\left(\frac{1}{7}\right) = \boxed{-1}$

b) $\left(\frac{1}{4}\right)^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

so $\left(\frac{1}{4}\right)^{-1/2} = \left(\frac{1}{2}\right)^{-1} = 2$

so $\log_{\frac{1}{4}}(2) = \boxed{-\frac{1}{2}}$

c) $a^1 = a$, so $\log_a(a) = \boxed{1}$

d) $e^{?}$ always has a positive value, so $\ln(0)$ is undefined.

e) Method 1

By properties of logarithms,

$\ln(e^{x^2-4}) = (x^2-4)\underbrace{\ln(e)}_{=1}$

$= x^2 - 4$

Method 2

$\ln(e^{x^2-4})$ asks the question

$e^{?} = e^{x^2-4}$.

So it's clear the answer is x^2-4.

g) $4^0 = 1$, so

$\log_4(1) = \boxed{0}$

h) $e^0 = 1$, so

$\ln(1) = \boxed{0}$

i) $4^1 = 4$, so

$\log_4(4) = \boxed{1}$

j) $e^1 = e$, so

$\ln(e) = \boxed{1}$

Property of logarithms

k) $\ln(4e) = \ln(4) + \underbrace{\ln(e)}_{=1}$

$= \ln(4) + 1$

(Cannot be further simplified)

l) $\boxed{\text{Cannot be further simplified}}$

m) Method 1

By the inverse relation,

n) This is undefined, since $\ln(x)$ is only defined for positive x-values.

o) This is undefined, since $\ln(x)$ is only defined for positive x-values.

p) $\log_2(4^x)$ is asking:

$2^{?} = 4^x$.

Notice that

$4^x = (2^2)^x$

$= 2^{2x}$

$e^{?} = e^{x^2-4}$.

So it's clear the answer is x^2-4.

f) $5^{2\log_5(3)} \overset{\text{Property of logarithms}}{=} 5^{\log_5(3^2)}$ Inverse relation

$= 3^2$

$= \boxed{9}$

m) Method 1

By the inverse relation,

$e^{\ln(1)} = \boxed{1}$

Method 2

$e^{\ln(1)} = e^0 = 1$

$4^x = (2^2)^x$

$= 2^{2x}$

so

$\log_2(4^x) = \boxed{2x}$

3. Solve for x:

(a) $e^x = 9$
(b) $e^x = -1$
(c) $2^x = 0$
(d) $2^x = \frac{1}{9}$
(e) $\log_x(4) = 9$
(f) $3^{x-5} = 4^x$
(g) $e^{4x+3} = 2$
(h) $2e^{3x+5} = 7$
(i) $3e^{x+1} = 2e^{2x}$

a) $\ln(e^x) = \ln(9)$

$\boxed{x = 9}$

b) e^x will always give a positive result. So no solution.

c) 2^x will always give a positive result. So no solution.

d) $\log_2(2^x) = \log_2\left(\frac{1}{9}\right)$

Inverse relation

$x = \log_2\left(\frac{1}{9}\right)$

or

$x = -\log_2(9)$

since $\frac{1}{9} = 9^{-1}$

e) $x^{\log_x(4)} = x^9$

Inverse relation

$4 = x^9$

$(4)^{1/9} = (x^9)^{1/9}$

$x = 4^{1/9}$ or $\sqrt[9]{4}$

f) When the bases aren't in common, apply $\ln(\ )$ to each side:

$\ln(3^{x-5}) = \ln(4^x)$

Property of logarithms

$(x-5)\ln(3) = x\ln(4)$

$x\ln(3) - 5\ln(3) = x\ln(4)$

$x\ln(3) - x\ln(4) = 5\ln(3)$

$x(\ln(3) - \ln(4)) = 5\ln(3)$

$$\boxed{x = \frac{5\ln(3)}{\ln(3)-\ln(4)}}$$

g) $\ln(e^{4x+3}) = \ln(2)$

$4x+3 = \ln(2)$

$$\boxed{x = \frac{\ln(2)-3}{4}}$$

h) $2e^{3x+5} = 7$

$e^{3x+5} = \frac{7}{2}$

$\ln(e^{3x+5}) = \ln\left(\frac{7}{2}\right)$

$3x+5 = \ln\left(\frac{7}{2}\right)$

$$\boxed{x = \frac{\ln\left(\frac{7}{2}\right)-5}{3}}$$

i) $\ln(3e^{x+1}) = \ln(2e^{2x})$

Property of logarithms

$\ln(3) + \ln(e^{x+1}) = \ln(2) + \underbrace{\ln(e^{(2x)})}_{\text{Inverse property}}$

$\ln(3) + (x+1)\underbrace{\ln(e)}_{=1} = \ln(2) + 2x$

$\ln(3) + x + 1 = \ln(2) + 2x$

$\ln(3) + 1 - \ln(2) = 2x - x$

$$\boxed{\begin{gathered} x = \ln(3) - \ln(2) + 1 \\ \text{or} \\ x = \ln\left(\tfrac{3}{2}\right) + 1 \end{gathered}}$$

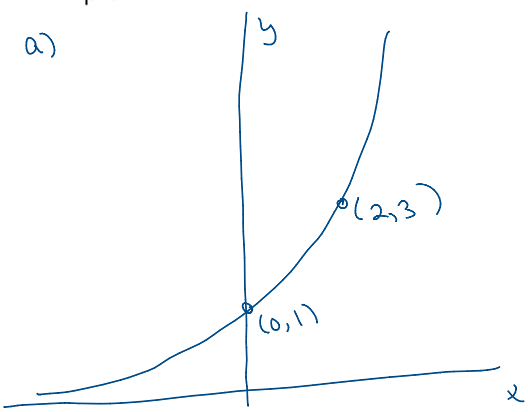
4. Suppose the graph of $y = a^x$ has the following properties:

- Passes through the point $(2, 3)$
- On the left-hand side of the graph (when x-is a large negative number), the y-values are small positive numbers

(a) Sketch the graph of $\log_a(x)$.

(b) What is the value of a?

a)



b) $\sqrt{3} = \sqrt{a^2}$

$a = \sqrt{3}$

(a has to be positive, so we don't include $-\sqrt{3}$ as a possible answer)

5. If $a^x = 3$, find the value of the following. (In some cases your answer will involve a).

(a) a^{3x}

$a^{3x} = (a^x)^3$

$= 3^3$

$= \boxed{27}$

(b) a^{x-1}

$a^{x-1} = \frac{a^x}{a^1} = \boxed{\frac{3}{a}}$

(c) a^{4x+1}

$a^{4x+1} = a^{4x} \cdot a^1$

$= (a^x)^4 \cdot a$

$= 3^4 \cdot a$

$= \boxed{81a}$

(d) $4a^{-\frac{x}{2}}$

$4a^{-x/2} = 4(a^x)^{-1/2}$

$= 4(3)^{-1/2}$

$= 4 \cdot \frac{1}{3^{1/2}}$

$= \boxed{\frac{4}{\sqrt{3}}}$ or $\frac{4\sqrt{3}}{3}$

6. Factor $e^{4x} - 4^{2x}$ as a difference of squares.

$e^{4x} - 4^{2x} = (e^{2x})^2 - (4^x)^2$

$= (e^{2x} - 4^x)(e^{2x} + 4^x)$

you can actually go a bit further:

$= (e^{2x} - (2^2)^x)(e^{2x} + 4^x)$

$= ((e^x)^2 - (2^x)^2)(e^{2x} + 4^x)$

$= \boxed{(e^x - 2^x)(e^x + 2^x)(e^{2x} + 4^x)}$