Math Vocabulary and Notation

Math Vocabulary

A **term** is either a single number or variable or the product of numbers and variables. Terms are separated by **addition** or **subtraction**.

Example In the equation $2x^2 - 3xe^x + 7x^2(x+5) = 6(x-3)$, there are three terms on the left side of the equals sign: $2x^2, 3xe^x$, and $7x^2(x+5)$. There is only one term on the right side of the equals sign: $6(x-3)$.

A **factor** is a single number or variable that divides another number or variable leaving no remainder. It is separated from another single number or variable by **multiplication**.

Example Consider $105x^3(x+3)^2(8x-3)$. This can be rewritten as $(1)(3)(5)(7)(x)(x)(x)(x+3)(x+3)(8x-3)$. So the **factors** of this **term** are $1, 3, 5, 15, 7, 21, 35, 105, x, x^2, x^3, x+3, (x+3)^2$, and $(8x-3)$.

A **product** is the result when two numbers or variables are multiplied together.

Example $3x^2$ is the product of 3 and x^2.

A **quotient** is the result when two numbers or variables are divided.

Example $\frac{3}{x^2}$ is the quotient of 3 and x^2, where 3 is the **numerator**, x^2 is the **denominator**.

A **rational number** is a number that can be made by dividing two integers, so of the form $\frac{a}{b}$ where both a and b are integers (whole numbers, not fractions or decimals).

Example $\frac{1}{3}, \frac{7}{6}$, and $2 = \frac{2}{1}$ are all rational numbers. $\sqrt{2}$ and e are not rational numbers.

A **polynomial** is a mathematical expression containing 1 or more terms, each of which is a constant multiplied by a variable raised to a power. The power of the variable must be a non-negative whole number, so no fractions or negative numbers.

Example The following are polynomials: 3, $2x^2+1$, $3x^4 - 6x^2$

The following are NOT polynomials: $6x^{-1} = \frac{6}{x}, 2\sqrt{x}+1, 3x^2+2x+6-5\sqrt[3]{x}$

Math Notation

Addition $a+b$ **Subtraction** $a-b$

Multiplication ab $a \cdot b$ $a \times b$ $(a)(b)$ $a * b$

the ones we use the most in calculus are ab and $(a)(b)$
So basically, no notation means you are MULTIPLYING.

Division a/b $\frac{a}{b}$ $a \div b$ $a : b$

the ones we use most in calculus are a/b and $\frac{a}{b}$

Roots $\sqrt{x} = x^{1/2}$ is the square root of x. It answers the question "what can be multiplied by itself 2 times to get x?"

$\sqrt[n]{x} = x^{1/n}$ is the n^{th} root of x. It answers the question "What can be multiplied by itself n times to get x?"

Powers a^n means to multiply a by itself n times

Functions $f(x)$ means to evaluate the function f at the value x. It reads f OF x.
We must be careful about the difference between OF and TIMES. See ☆.

☆ It is important to be careful with $a(b)$
If both a and b are numbers or variables, then this represents multiplication and reads a TIMES b.
If, however, a is a function, it reads a OF b and means to evaluate the FUNCTION a at the VALUE b.

Equals $a=b$ means that a and b are THE SAME THING. It cannot be used for anything else, so be VERY careful where you put =. If the two things are not the same, do not put it!

Comparison
$a > b$ a is bigger than b
$a < b$ a is smaller than b
$a \geq b$ a is bigger than OR equal to b
$a \leq b$ a is smaller than OR equal to b

REMEMBER! the mouth will always be eating the BIGGER value

Order of Operations

Now let's remember the order of operations: PEDMAS
P = Parentheses
E = Exponents (powers, roots)
M = Multiplication
D = Division
A = Addition
S = Subtraction

I was always taught to think **Please Excuse My Dear Aunt Sally** when solving any math problem. You may have also seen this with a **B** for brackets instead of a **P**.
The order of the letters tells us the order in which we should simplify things; it helps us figure out which operation to do first.

For the following examples, the goal will be to determine which operation should be done first.

Example 1 In $7x - 6$, there are two operations: Multiplication and Subtraction. As Multiplication comes before Subtraction, we would start by multiplying the 7 times the x value.

Example 2 Given $7 - 3(x + 2)$, we would need to start by dealing with the Parentheses. For this, we have two options. We could get rid of the Parentheses by Adding together the x value and the 2 OR we would do Multiplication to **distribute** the -3 inside the parentheses, resulting in $7 - 3x - 6$.

Example 3 In $(4^2)(2) + (3)(5)$, we see Exponents, Multiplication, and Addition. Exponents come first, so we would start by evaluating the 4^2 to get 16 before proceeding to the Multiplication.

Example 4
WARNING! Given something like $\frac{x+2}{2x}$, it is important to recognize that there are **IMPLIED PARENTHESES** around the $x + 2$ and the $2x$, or the numerator and denominator. This is equivalent to $(x + 2) \div (2x)$. This is **NOT** equivalent to $x + 2 \div 2x$. The first thing we should do is add the x value and the 2 in the numerator AND multiply the 2 and the x in the denominator.
Because of the **IMPLIED PARENTHESES** , both of the following are **INCORRECT**.
WRONG: $\frac{x+2}{2x} = \frac{\cancel{x}+2}{2\cancel{x}}$
WRONG: $\frac{x+2}{2x} = \frac{x+\cancel{2}}{\cancel{2}x}$
In both cases, this is equivalent to doing **D**ivision before doing **P**arentheses. Big no no!

Translating Math Into Words

Keeping all of this in mind can sometimes be challenging, so a good way to think about any math problem is to try to convert the symbols into words. What is the notation actually asking you to do. So with that in mind, let's translate some math problems into words.

Exercise 1 Translate $7x - 6$ into words.

Solution: The $7x$ means to multiply the 7 times the x. As multiplication comes before subtraction, we would first multiply 7 times x and then subtract 6. So we can think of this as 6 **less than** 7 **times** x or 7 **times** x **decreased by** 6.

Exercise 2 Translate $\frac{x}{y} + 4$ into words.

Solution: $\frac{x}{y}$ means that we should divide x by y, or do the quotient of x and y. Because division comes before addition, we would first do the division and then add 4. So we can think of this as x **divided by** y **increased by** 4 or 4 **more than** x **divided by** y.

Exercise 3 Translate $5(2x + 3)$ into words.

Solution: The parentheses here tell us that we should do the addition BEFORE the multiplication. In the parentheses, we multiply the 2 times the x before doing the addition. Even though the 5 is not also in parentheses, 5 is a value, not a function, so we recognize that we are multiplying the 5 TIMES the $2x + 3$. So we can think of this as 3 **more than** 2 **times** x **all times** 5 or 5 **times the value** 2 **times** x **increased by** 3.

Exercise 4 Translate $\frac{x+4}{2}$ into words

Solution: This one is a little more challenging. Division comes before addition, so it seems like maybe we should do that first. BUT, when you have multiple **terms** in the numerator, there are implied parentheses. Meaning that this is the same as $\frac{(x+4)}{2}$, so we need to do the addition BEFORE the division because parentheses come first. So we can think of this as 4 **more than** x **all divided by** 2 or **the quotient of** x **increased by** 4 **and** 2.

Translating Words to Math

Exercise 1 The sum of two numbers is 47. The larger one is seven more than four times the smaller one. Set-up a system of equations that represents this situation and then write one equation that will allow you to solve for one of the variables.

Solution: We have two numbers, and we don't know what they are. When we don't know what something is, we assign it a variable. So let's let x =bigger number and y =smaller number.
Sum means addition, so we know that we should have $x + y = 47$.
The larger one is x and it should be 7 more than (this means addition) 4 times (this means multiplication) the smaller one, y, giving us $x = 4y + 7$.
So $x+y = 47$ AND $x = 4y+7$ must both be true at the same time. Since we want just ONE equation, we can replace the x in the original equation to get one equation in terms of y:
$4y + 7 + y = 47$ gives $5y = 50$.

Exercise 2 What three consecutive odd integers have a sum of 33?

Solution: Consecutive integers means that each one is just 1 bigger than the one before it. So, for example, $2, 3, 4$ would be three consecutive integers. In this case, though, we want three consecutive **odd** integers, so that would be like $1, 3, 5$, meaning that each one is 2 greater than the previous. Let x represent the smallest of the three integers. Then $x + 2$ would be the second of the integers and $x + 2 + 2 = x + 4$ would be the third.
Their sum should be 33, and sum means addition, so this gives us $x + x + 2 + x + 4 = 33$, which we can easily solve.
First, we combine like terms to get $3x + 6 = 33$.
Then, we subtract 6 to get $3x = 27$.
Lastly, we divide by 3 to get $x = 9$. So the smallest of our three integers is 9. The three integers will thus be $9, 11$, and 13.

Exercise 3 Find a rational number that is one-half the difference between 11 and 8.

Solution: Remember that rational means the number can be given as a fraction and difference means subtraction. In this case, half the difference means we MULTIPLY the difference by $\frac{1}{2}$. So our number will be
$x = \frac{1}{2}(11 - 8) = \frac{1}{2}(3) = \frac{3}{2}$.
We were asked for a rational number, so under no circumstances should we write this as a decimal.