Solutions to Trigonometry Problem Set

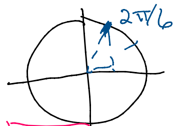
For a video version of these solutions, follow this link: https://youtu.be/GW6DAkKCeAE

1. **Without the help of a calculator or notes, find the following values:**

(a) $\sin(\pi/3)$ (c) $\cot(\pi/2)$ (e) $\tan(5\pi/3)$

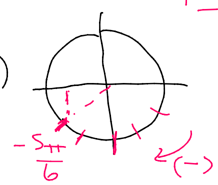
(b) $\csc(-5\pi/6)$ (d) $\cos(9\pi/4)$ (f) $\sin^3(5\pi/4)(\sec^2(\pi/3) - \csc^2(\pi/3))$

a) $\frac{\pi}{3} = \frac{2\pi}{6}$



$\sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

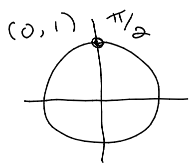
b)



At this position, y is negative and smaller than x, so $\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$

$\csc\left(-\frac{5\pi}{6}\right) = \frac{1}{\sin\left(-\frac{5\pi}{6}\right)} = \boxed{-2}$

c)

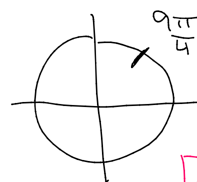


$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

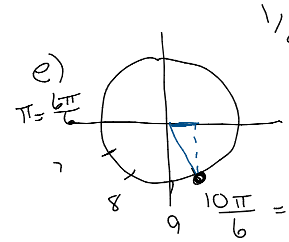
$\cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$

d)



$\cos\left(\frac{9\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

e)



x is positive and smaller than y

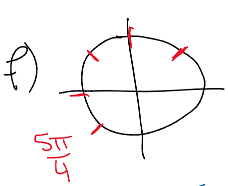
y is negative and larger than x

so $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$,

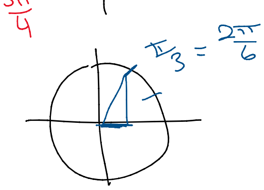
$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\tan\left(\frac{5\pi}{3}\right) = \frac{\sin\left(\frac{5\pi}{3}\right)}{\cos\left(\frac{5\pi}{3}\right)} = \frac{-\sqrt{3}/2}{1/2} = \boxed{-\sqrt{3}}$

f)



$\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \rightarrow \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \rightarrow \sec\left(\frac{\pi}{3}\right) = \frac{1}{1/2} = 2$

$$\sin^3\left(\frac{5\pi}{4}\right)\left(\sec^2\left(\frac{\pi}{3}\right) - \csc^2\left(\frac{\pi}{3}\right)\right) = \left(\sin\left(\frac{5\pi}{4}\right)\right)^3 \cdot \left(\left(\sec\left(\frac{\pi}{3}\right)\right)^2 - \left(\csc\left(\frac{\pi}{3}\right)\right)^2\right)$$

$$= \left(-\frac{\sqrt{2}}{2}\right)^3 \cdot \left((2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2\right)$$

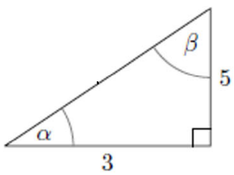
$$= -\frac{2\sqrt{2}}{8} \cdot \left(4 - \frac{4}{3}\right)$$

$$= -\frac{\sqrt{2}}{4} \cdot \left(\frac{12}{3} - \frac{4}{3}\right)$$

$$= -\frac{\sqrt{2}}{4} \cdot \left(\frac{12}{3} - \frac{4}{3}\right)$$

$$= -\frac{\sqrt{2}}{\cancel{4}} \cdot \left(\frac{\cancel{8}^{2}}{3}\right) = \boxed{\frac{-2\sqrt{2}}{3}}$$

2. Given the following right triangle:



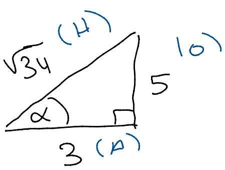
(a) Find the length of the missing side

(b) Find $\cot(\alpha)$

(c) Find $\sec(\beta)$

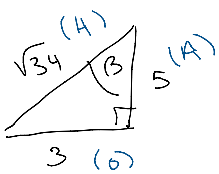
a) $5^2 + 3^2 = H^2 \rightarrow H = \sqrt{25+9} = \boxed{\sqrt{34}}$

b)



$\cot(\alpha) = \frac{1}{\tan(\alpha)} = \frac{A}{O} = \boxed{\frac{5}{3}}$

c)



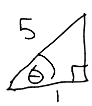
$\sec(\beta) = \frac{1}{\cos(\beta)} = \frac{H}{A} = \boxed{\frac{\sqrt{34}}{5}}$

3. If $\cos(\theta) = \frac{1}{5}$, what is the value of:

(a) $\sin(\theta)$ (b) $\sec(\theta)$ (c) $\tan(\theta)$

a) Method 1:

$\cos(\theta) = \frac{A}{H}$ so we can draw



$\leftarrow O = \sqrt{5^2 - 1^2} = \sqrt{24} = 2\sqrt{6}$

$\sin(\theta) = \frac{O}{H} = \boxed{\frac{2\sqrt{6}}{5}}$

a) Method 2:

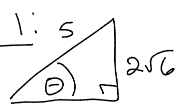
$\sin^2\theta + \cos^2\theta = 1$

so

$\sin^2\theta + \left(\frac{1}{5}\right)^2 = 1$

$\sin\theta = \sqrt{1 - \frac{1}{25}} = \sqrt{\frac{24}{25}} = \boxed{\frac{2\sqrt{6}}{5}}$

b) Method 1:



$\rightarrow \sec\theta = \frac{H}{A} = \frac{5}{1} = \boxed{5}$

Method 2:

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/5} = \boxed{5}$

c) Method 1:

$\rightarrow \tan\theta = \frac{O}{A} = \frac{2\sqrt{6}}{1} = \boxed{2\sqrt{6}}$

Method 2:

$\sin^2\theta + \cos^2\theta = 1$

$\sin\theta = \sqrt{1 - \left(\frac{1}{5}\right)^2}$

$\sin\theta = \frac{\sqrt{24}}{5}$

divide by $\cos\theta = \frac{1}{5}$

$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{\sqrt{24}}{5}\right)}{}$

divide by $\cos\theta = \frac{1}{5}$

$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{\sqrt{24}}{5}\right)}{1/5}$$

$$\tan\theta = \sqrt{24} = \boxed{2\sqrt{6}}$$

4. Use trigonometric identities to simplify as much as possible:

(a) $\cot(x)\sec(x)$

(b) $\tan(\theta)\cos(\theta)$

(c) $\frac{1-\sin^2(x)}{1-\cos^2(x)}$

(d) $\cot(y)\tan(y)$

(e) $\dfrac{\sec(\theta)\sin^2(\theta)}{1+\sec(\theta)}$

a) $\cot(x)\cdot\sec(x) = \frac{\cancel{\cos}(x)}{\sin(x)}\cdot\frac{1}{\cancel{\cos}(x)} = \frac{1}{\sin(x)} = \boxed{\csc(x)}$ (if $\cos\theta \neq 0$)

b) $\tan\theta\cdot\cos\theta = \frac{\sin\theta}{\cancel{\cos}\theta}\cdot\cancel{\cos}\theta = \boxed{\sin\theta}$ (if $\cos\theta \neq 0$)

c) $\frac{1-\sin^2 x}{1-\cos^2(x)} = \frac{\cos^2(x)}{\sin^2(x)} = \left(\frac{\cos(x)}{\sin(x)}\right)^2 = \boxed{\cot^2(x)}$

d) $\cot(y)\cdot\tan(y) = \frac{\cancel{\cos}(y)}{\cancel{\sin}(y)}\cdot\frac{\cancel{\sin}(y)}{\cancel{\cos}(y)} = \boxed{1}$ (if $\sin(y)$ and $\cos(y) \neq 0$)

e) $\dfrac{\sec\theta\cdot\sin^2\theta}{1+\sec\theta} = \dfrac{\left(\frac{1}{\cos\theta}\cdot\sin^2\theta\right)\cdot\cos\theta}{\left(1+\frac{1}{\cos\theta}\right)\cdot\cos\theta}$

$= \boxed{\dfrac{\sin^2\theta}{\cos\theta+1}}$ Pretty good, but we can do a bit more!

Difference of squares! $\rightarrow$ $= \dfrac{1-\cos^2(\theta)}{1+\cos\theta} = \dfrac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)}$ $=1$ (if $1+\cos\theta \neq 0$)

$= \boxed{(1-\cos\theta)}$ (if $1+\cos\theta \neq 0$)

5. Solve for x over the specified interval:

(a) $2\cos(x)+2=1$, on $[0,2\pi)$

(b) $6\sin(x)=\sqrt{18}$, on $[0,2\pi)$

(c) $1+\sin(x)=1-\cos(x)$, on $[0,2\pi)$

(d) $\tan(3x)=\sqrt{3}$, on $[0,\pi]$

(e) $6\csc(x-\frac{\pi}{3})=12$, on $[-\pi/2,\pi/2]$

(f) $\sin^2(x)=\frac{1}{2}$, on $[0,2\pi)$

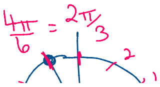
(g) $\sin^2(x)-2\cos(x)=\cos^2(x)-\cos(x)$, on $[0,2\pi)$

☆ First solve for the value of the trig function, then use knowledge of the unit circle ☆

a) $2\cos(x)+2=1$

$2\cos(x) = -1$

$\frac{4\pi}{6} = \frac{2\pi}{3}$



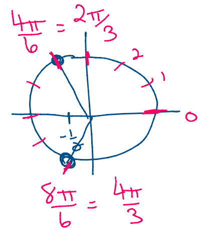
b) $6\sin(x)=\sqrt{18}$

$\sin(x) = \frac{\sqrt{9\cdot 2}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

a) $2\cos(x) + 2 = 1$

$2\cos(x) = -1$

$\cos(x) = -\frac{1}{2}$ →

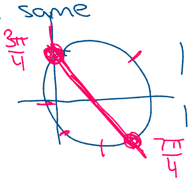


$\boxed{x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}}$

c) $1 + \sin(x) = 1 - \cos(x)$

$\sin(x) = -\cos(x)$

↳ sin and cos need to be the same number but opposite signs: $\frac{3\pi}{4}$



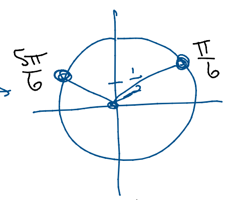
$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

e) $6\csc\left(x - \frac{\pi}{3}\right) = 12$

$\csc\left(x - \frac{\pi}{3}\right) = 2$

$\frac{1}{\sin\left(x - \frac{\pi}{3}\right)} = 2$

$\sin\left(x - \underbrace{\frac{\pi}{3}}_{= \frac{2\pi}{6}}\right) = \frac{1}{2}$ →



Sin is $\frac{1}{2}$ at $\frac{\pi}{6}$ and $\frac{5\pi}{6}$, and at equivalent angles: $-\frac{11\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}, \ldots$

and $-\frac{7\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{6}, \ldots$

Solve for x: $\left(x - \frac{2\pi}{6}\right) = -\frac{11\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}, \ldots$

and $-\frac{7\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{6}$

so $x = \underbrace{-\frac{9\pi}{6}}_{-\frac{3\pi}{2}}, \underbrace{\frac{3\pi}{6}}_{\pi/2}, \frac{15\pi}{6}, \ldots$

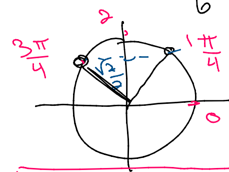
and $-\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{19\pi}{6}, \ldots$

Now, take the solutions that are in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. There is only one: $\boxed{x = \frac{\pi}{2}}$

g) The trick with a problem like this one is to use the Pythagorean identity to convert everything to sin or to cos, and then treat it like a quadratic:

b) $6\sin(x) = \sqrt{18}$

$\sin(x) = \frac{\sqrt{9 \cdot 2}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$



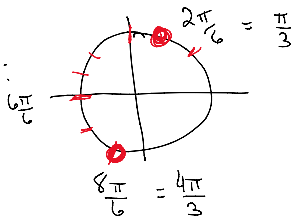
$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$

d) First we have to solve for the possible values of 3x.

$\tan(3x) = \sqrt{3} = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}/2}{1/2}$ ← sin, ← cos $\left(\tan = \frac{\sin}{\cos}\right)$

tan gives $\sqrt{3}$ if $\sin = \frac{\sqrt{3}}{2}$, $\cos = \frac{1}{2}$ OR if $\sin = -\frac{\sqrt{3}}{2}$, $\cos = -\frac{1}{2}$

so at:



so

$\tan(3x) = \sqrt{3}$ at $3x = \frac{\pi}{3}, \frac{4\pi}{3}$, or any equivalent angle:

$3x = \frac{\pi}{3}, \frac{7\pi}{3}, \frac{13\pi}{3}, \ldots$

and $\frac{4\pi}{3}, \frac{10\pi}{3}, \frac{16\pi}{3}, \ldots$

Finally, solve for x in these solutions, and take the values that are in $[0, \pi)$:

$x = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}, \ldots$

and $\frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}, \ldots$

} Answer: $\boxed{x = \frac{\pi}{9}, \frac{4\pi}{9} \text{ or } \frac{7\pi}{9}}$

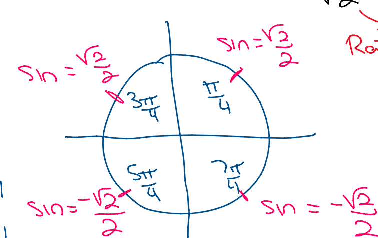
f) $\sin^2(x) = \frac{1}{2}$

$(\sin(x))^2 = \frac{1}{2}$

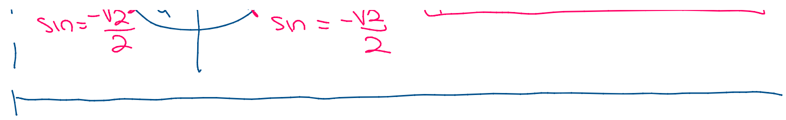
$\sin(x) = \pm\sqrt{\frac{1}{2}}$

$= \pm\frac{1}{\sqrt{2}} = \boxed{\pm\frac{\sqrt{2}}{2}}$

Rationalize



→ $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}}$



everything to sin or to cos, and then treat it like a quadratic:

$$\underbrace{\sin^2(x)}_{=(1-\cos^2(x))} - 2\cos(x) = \cos^2(x) - \cos(x)$$

convert to only cos

$$1 - \cos^2(x) - 2\cos(x) = \cos^2(x) - \cos(x)$$

Move all to one side

$$0 = 2\cos^2(x) + 2\cos(x) - \cos(x) - 1$$

$$0 = 2\cos^2(x) + \cos(x) - 1$$

Factor

Now we have a quadratic which can be factored. If it helps, you can let $\cos(x) = y$ and temporarily write it as $0 = 2y^2 + y - 1$

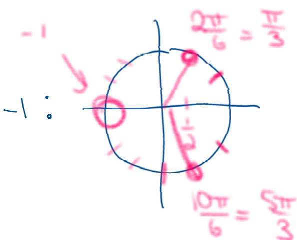
$$0 = (2\cos(x) - 1)(\cos(x) + 1)$$

set each factor to 0

$$2\cos(x) - 1 = 0 \quad \text{or} \quad \cos(x) + 1 = 0$$

$$\cos(x) = \frac{1}{2} \qquad\qquad \cos(x) = -1$$

Finally, find the angles in $[0, 2\pi)$ for which $\cos = \frac{1}{2}$ or -1:



$\cos = -1$ at π

$\cos = \frac{1}{2}$ at $\frac{\pi}{3}, \frac{5\pi}{3}$

so

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} \text{ or } \pi}$$