201-203-RE - Practice Set #6: Evaluating Definite Integrals (Including Substitutions)

Use the fundamental theorem of calculus to evaluate the following integrals.

(1) $\displaystyle\int_1^4 \left(\frac{2}{x\sqrt{x}} - 5\sqrt{x}\right)\,dx$

(2) $\displaystyle\int_1^8 \left(6\sqrt[3]{x} - \frac{8}{\sqrt[3]{x^2}}\right)\,dx$

(3) $\displaystyle\int_{-1}^0 \sqrt[3]{x}(3-x^2)\,dx$

(4) $\displaystyle\int_{\sqrt{22}}^{\sqrt{3}} \frac{x\,dx}{\sqrt[3]{x^2+5}}$

(5) $\displaystyle\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

(6) $\displaystyle\int_0^7 \sqrt{9+x}\,dx$

(7) $\displaystyle\int_{-8}^1 x^3(4-\sqrt[3]{x})\,dx$

(8) $\displaystyle\int_1^2 \frac{x+1}{\sqrt{x^2+2x}}\,dx$

(9) $\displaystyle\int_0^1 (x-1)e^{x^2-2x}\,dx$

(10) $\displaystyle\int_1^2 \frac{6+x^6}{x^2}\,dx$

(11) $\displaystyle\int_1^3 \frac{x^2}{1+x^3}\,dx$

(12) $\displaystyle\int_0^1 \frac{6x}{(1+x^2)^2}\,dx$

(13) $\displaystyle\int_0^2 (x^3-3x)^3(x^2-1)\,dx$

(14) $\displaystyle\int_0^1 e^{x^2-2x}(1-x)\,dx$

(15) $\displaystyle\int_0^{\pi/2} \frac{3\sin(x)}{9\cos(x)+1}\,dx$

(16) $\displaystyle\int_0^1 \frac{9(2+e^{-3x})^2}{e^{3x}}\,dx$

(17) $\displaystyle\int_0^{\pi/4} \sec^2(x)e^{1+2\tan(x)}\,dx$

(18) $\displaystyle\int_0^1 (9x-3)e^{3x^2-2x}\,dx$

(19) $\displaystyle\int_{-1}^0 \frac{12(x+e^{3x})}{(3x^2+2e^{3x})^2}\,dx$

(20) $\displaystyle\int_0^{\pi/2} \frac{14\sin(x)}{\sqrt{9+7\cos(x)}}\,dx$

(21) $\displaystyle\int_1^9 \frac{6e^{1+\sqrt{x}}}{\sqrt{x}}\,dx$

(22) $\displaystyle\int_0^1 9e^{3x}\sqrt{e^{3x}+3}\,dx$

(23) $\displaystyle\int_0^2 18x^2\sqrt{9-x^3}\,dx$

(24) $\displaystyle\int_0^1 56x^3(x^4-1)^6\,dx$

(25) $\displaystyle\int_0^1 10x\sqrt[3]{(1-x^2)^2}\,dx$

(26) $\displaystyle\int_0^1 \frac{6(4-x)}{(-1-8x+x^2)^{2/3}}\,dx$

(27) $\displaystyle\int_0^1 \frac{9x^3-3x^2-2x-5}{3x+1}\,dx$

(28) $\displaystyle\int_0^1 \frac{16x^3}{(1+x^4)^2}\,dx$

(29) $\displaystyle\int_0^2 \frac{6x^2}{9-x^3}\,dx$

(30) $\displaystyle\int_0^{\pi/2} 6\cos(x)e^{1-2\sin(x)}\,dx$

(31) $\displaystyle\int_0^1 \frac{6x^3-5x^2-4x-2}{2x+1}\,dx$

(32) $\displaystyle\int_0^1 \left(\frac{4\ln(x+1)}{x+1} + 2e^{2x}\right)dx$

(33) $\displaystyle\int_{-1}^0 \frac{24(-2x+x^3)}{(1+4x^2-x^4)^2}\,dx$

(34) $\displaystyle\int_1^2 \frac{9(x^2+2)}{x^3+6x}\,dx$

(35) $\displaystyle\int_0^3 \frac{18e^{3x}}{e^{3x}+2}\,dx$

(36) $\displaystyle\int_0^2 \frac{6x^2+13x-13}{x+3}\,dx$

(37) $\displaystyle\int_{-1}^0 \frac{9x^3+10x^2-16x+3}{x+2}\,dx$

(38) $\displaystyle\int_0^{\ln(2)} \frac{6(e^{3x}+1)}{e^{3x}+3x}\,dx$

(39) $\displaystyle\int_2^3 \frac{2(x^3-1)}{(2x^4-8x)^2}\,dx$

(40) $\displaystyle\int_3^4 \frac{-8x^2+21x-12}{2-x}\,dx$

(41) $\displaystyle\int_1^2 \frac{4x^3e^{x^2-1}+6}{x^2}\,dx$

(42) $\displaystyle\int_1^e \frac{(3+2\ln(x))^2}{4x}\,dx$

(43) $\displaystyle\int_1^e \frac{6}{x(4+3\ln(x))^2}\,dx$

(44) $\displaystyle\int_{-1}^0 \frac{\ln(\sqrt{3x+4})}{3x+4}\,dx$

(45) $\displaystyle\int_0^1 \frac{6x^2}{e^{x^3-1}}\,dx$

(46) $\displaystyle\int_{-\pi/6}^0 \ln[\cos^3(2x)]\tan(2x)\,dx$

(47) $\displaystyle\int_1^e \frac{e^{1+\ln(x)}}{x}\,dx$

(48) $\displaystyle\int_0^{\ln(2)} \frac{\ln(1+e^x)}{1+e^x}e^x\,dx$

(49) $\displaystyle\int_0^2 x\sqrt{4x+1}\,dx$

(50) $\displaystyle\int_0^1 \frac{x^2}{2-x}\,dx$

(51) $\displaystyle\int_{-2}^1 \frac{2x+1}{(x+3)^2}\,dx$

(52) $\displaystyle\int_0^7 \frac{19-x^2}{(x+1)^{5/3}}\,dx$

(53) $\displaystyle\int_0^{\ln(2)} \frac{e^{2x}\left(e^{2x}-4\right)}{e^{2x}+4}\,dx$

(54) $\displaystyle\int_1^{e^3} \frac{\ln(x)+3}{x\sqrt{\ln(x)+1}}\,dx$

(55) $\displaystyle\int_3^4 \left(\frac{6}{(x-2)^3} + \frac{5}{(x-2)^2} - \frac{2}{x-2}\right)dx$

ANSWERS:

(1) $\frac{-64}{3}$

(2) $\frac{87}{2}$

(3) $\frac{-39}{20}$

(4) $\frac{-15}{4}$

(5) $2(e^2 - e)$

(6) $\frac{74}{3}$

(7) $\frac{-77814}{13}$

(8) $2\sqrt{2} - \sqrt{3}$

(9) $\frac{1}{2}(e^{-1} - 1)$

(10) $\frac{46}{5}$

(11) $\frac{1}{3}\ln(14)$

(12) $\frac{3}{2}$

(13) $\frac{4}{3}$

(14) $\frac{1}{2}(1 - e^{-1})$

(15) $\frac{1}{3}\ln(10)$

(16) $27 - (2 + e^{-3})^3$

(17) $\frac{1}{2}(e^3 - e)$

(18) $\frac{3}{2}(e - 1)$

(19) $-1 + \frac{2}{3 + 2e^{-3}}$

(20) 4

(21) $12(e^4 - e^2)$

(22) $2(e^3 + 3)^{3/2} - 16$

(23) 104

(24) 2

(25) 3

(26) 9

(27) $-\frac{10}{3}\ln(2)$

(28) 2

(29) $4\ln(3)$

(30) $3(e - e^{-1})$

(31) $-1 - \ln(3)$

(32) $2(\ln(2))^2 + e^2 - 1$

(33) $\frac{9}{2}$

(34) $3\ln\left(\frac{20}{7}\right)$

(35) $6\ln\left(\frac{e^9 + 2}{3}\right)$

(36) $2\ln\left(\frac{5}{3}\right) + 2$

(37) $7 + 3\ln(2)$

(38) $2\ln(8 + 3\ln(2))$

(39) $\frac{61}{4416}$

(40) $23 + 2\ln(2)$

(41) $2e^3 + 1$

(42) $\frac{49}{12}$

(43) $\frac{3}{14}$

(44) $\frac{1}{12}(\ln(4))^2$

(45) $2e - 2$

(46) $\frac{3}{4}(\ln(2))^2$

(47) $e^2 - e$

(48) $\frac{1}{2}(\ln(3))^2 - \frac{1}{2}(\ln(2))^2$

(49) $\frac{27}{10}$

(50) $\frac{11}{2} + \ln(8)$

(51) $2\ln(4) - \frac{15}{4}$

(52) $-\frac{51}{2}$

(53) $\frac{3}{2} + 4\ln\left(\frac{5}{8}\right)$

(54) $\frac{26}{3}$

(55) $\frac{19}{4} - 2\ln(2)$