Make a variation table for the following functions, and find the local extrema.

(1) $f(x) = -\dfrac{x^3}{3} + x^2 + 3x + 4$

(2) $f(x) = \dfrac{5x^2 + 5}{x}$

(3) $f(x) = \dfrac{-3x^2 - 12}{x}$

(4) $f(x) = \dfrac{1}{4}x^4 + \dfrac{1}{3}x^3 - x^2 + 4$

(5) $f(x) = \dfrac{3x^2 - 5x + 27}{x}$

(6) $f(x) = \dfrac{x^2 - 2x + 9}{2 - x}$

(7) $f(x) = \dfrac{1}{2}x^4 + 2x^3 + 2$

(8) $f(x) = \dfrac{4x^2 + 9x + 9}{x + 1}$

(9) $f(x) = \dfrac{2x^3 - 4}{x}$

(10) $f(x) = -\dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 + 6x + 3$

(11) $f(x) = -\dfrac{6x^2 + 24}{x}$

(12) $f(x) = \dfrac{-5x^2 + 2x + 8}{x^2}$

(13) $f(x) = \dfrac{1}{4}x^4 - \dfrac{5}{3}x^3 + 2x^2 + 3$

(14) $f(x) = \dfrac{-2x^2 + 3x - 8}{x}$

(15) $f(x) = \dfrac{x^2 - x + 4}{x - 1}$

(16) $f(x) = \dfrac{3}{4}x^4 - 3x^3 + 4$

(17) $f(x) = \dfrac{2x^2 + 7x + 8}{x + 2}$

(18) $f(x) = \dfrac{3x^3 + 6}{x}$

(19) $f(x) = \dfrac{x^3}{x + 2}$

Find the absolute extrema of the function on the given interval.

(20) $f(x) = \dfrac{1}{2}x^4 - 4x^2 + 5$ on $[1, 3]$

(21) $f(x) = \dfrac{-x^3 - 4}{x^2}$ on $[1, 4]$

(22) $f(x) = \dfrac{5}{2}x^4 - \dfrac{20}{3}x^3 + 6$ on $[-1, 3]$

(23) $f(x) = \dfrac{3}{2}x^4 - 4x^3 + 4$ on $[0, 3]$

(24) $f(x) = 2x^4 - 36x^2 + 20$ on $[-4, -1]$

(25) $f(x) = \dfrac{2x^3 + 27}{2x^2}$ on $[2, 5]$

(26) $f(x) = \dfrac{40}{3}x^3 - 2x^4 + 10$ on $[-1, 6]$

(27) $f(x) = -\dfrac{4}{5}x^5 + \dfrac{1}{2}x^4 + 8$ on $[-2, 1]$

(28) $f(x) = \dfrac{x^2 + 25}{4x}$ on $[2, 6]$

(29) $f(x) = x^4 - 8x^2$ on $[-2, 3]$

ANSWERS:

(1)

x	$-\infty$		-1		3		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$	

local min: $\left(-1, \dfrac{7}{3}\right)$ local max: $(3, 13)$

(2)

x	$-\infty$		-1		0		1		$+\infty$
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(1, 10)$ local max: $(-1, -10)$

(3)

x	$-\infty$		-2		0		2		$+\infty$
$f'(x)$		$-$	0	$+$		$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$	

local min: $(-2, 12)$ local max: $(2, -12)$

(4)

x	$-\infty$		-2		0		1		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$		$\nearrow$	

local min: $\left(-2, \frac{4}{3}\right)$ and $\left(1, \frac{43}{12}\right)$

local max: $(0, 4)$

(5)

x	$-\infty$		-3		0		3		$+\infty$
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(3, 13)$ local max: $(-3, -23)$

(6)

x	$-\infty$		-1		2		5		$+\infty$
$f'(x)$		$-$	0	$+$		$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$	

local min: $(-1, 4)$ local max: $(5, -8)$

(7)

x	$-\infty$		-3		0		$+\infty$
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$	

local min: $\left(-3, -\frac{23}{2}\right)$ local max: none

(8)

x	$-\infty$		-2		-1		0		$+\infty$
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(0, 9)$ local max: $(-2, -7)$

(9)

x	$-\infty$		$\cancel{0}$ -1		$\cancel{1}$ 0		$+\infty$
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$		$\searrow$		$\cancel{\searrow}$ $\nearrow$		$\nearrow$	

local min: ~~$(1, -2)$~~ $(-1, 6)$ local max: none

(10)

x	$-\infty$		-3		2		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$	

local min: $\left(-3, -\frac{21}{2}\right)$ local max: $\left(2, \frac{31}{3}\right)$

(11)

x	$-\infty$		-2		0		2		$+\infty$
$f'(x)$		$-$	0	$+$		$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$	

local min: $(-2, 24)$ local max: $(2, -24)$

(12)

x	$-\infty$		-8		0		$+\infty$
$f'(x)$		$-$	0	$+$		$-$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$	

local min: $\left(-8, -\frac{41}{8}\right)$ local max: none

(13)

x	$-\infty$		0		1		4		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$		$\nearrow$	

local min: $(0, 3)$ and $\left(4, -\frac{23}{3}\right)$

local max: $\left(1, \frac{43}{12}\right)$

(14)

x	$-\infty$		-2		0		2		$+\infty$
$f'(x)$		$-$	0	$+$		$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$	

local min: $(-2, 11)$ local max: $(2, -5)$

(15)

x	$-\infty$		-1		1		3		$+\infty$
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(3, 5)$ local max: $(-1, -3)$

(16)

x	$-\infty$		0		3		$+\infty$
$f'(x)$		$-$	0	$-$	0	$+$	
$f(x)$		$\searrow$		$\searrow$		$\nearrow$	

local min: $\left(3, -\dfrac{65}{4}\right)$ local max: none

(17)

x	$-\infty$		-3		-2		-1		$+\infty$
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(-1, 3)$ local max: $(-3, -47)$

(18)

x	$-\infty$		0		1		$+\infty$
$f'(x)$		$-$		$-$	0	$+$	
$f(x)$		$\searrow$		$\searrow$		$\nearrow$	

local min: $(1, 9)$ local max: none

(19)

x	$-\infty$		$\cancel{-4}$ -3		-2		0		$+\infty$
$f'(x)$		$-$	0	$+$		$+$	0	$+$	
$f(x)$		$\searrow$		$\nearrow$		$\nearrow$		$\nearrow$	

local min: $\cancel{(-4, 32)}$ $(-3, 27)$ local max: none

(20) Abs. min.: -3, at $x = 2$; Abs. max.: $\dfrac{19}{2}$ at $x = 3$

(21) Abs. min.: -5, at $x = 1$; Abs. max.: -3 at $x = 2$

(22) Abs. min.: $-\dfrac{22}{3}$, at $x = 2$; Abs. max.: $\dfrac{57}{2}$ at $x = 3$

(23) Abs. min.: -4, at $x = 2$; Abs. max.: $\dfrac{35}{2}$ at $x = 3$

(24) Abs. min.: -142, at $x = -3$; Abs. max.: -14 at $x = -1$

(25) Abs. min.: $\dfrac{9}{2}$, at $x = 3$; Abs. max.: $\dfrac{277}{50}$ at $x = 5$

(26) Abs. min.: $-\dfrac{16}{3}$, at $x = -1$; Abs. max.: $\dfrac{1280}{3}$ at $x = 5$

(27) Abs. min.: $\dfrac{77}{10}$, at $x = 1$; Abs. max.: $\dfrac{208}{5}$ at $x = -2$

(28) Abs. min.: $\dfrac{5}{2}$, at $x = 5$; Abs. max.: $\dfrac{29}{8}$ at $x = 2$

(29) Abs. min.: -16, at $x = -2$ and $x = 2$; Abs. max.: 9 at $x = 3$