

Determine:

$$(1) \int (3x+4)^3 x \, dx ; (2) \int \frac{(x+1) \, dx}{\sqrt{x-1}} ; (3) \int \sqrt{1-2x} x^2 \, dx$$

$$(4) \int \sqrt[3]{5+4x} (3x-2) \, dx ; (5) \int \frac{2x-1}{\sqrt{x+3}} \, dx ; (6) \int \frac{x^2-1}{\sqrt{2x-1}} \, dx$$

$$(7) \int_{-2}^6 x^2 (x+2)^{1/3} \, dx ; (8) \int \sqrt[3]{(3x^2-4)^2} x^3 \, dx ; (9) \int \frac{x^5 \, dx}{(5x^3-7)^2}$$

Answers:

$$(1) \frac{1}{45} (3x+4)^5 - \frac{1}{9} (3x+4)^4 + C ; (2) \frac{2}{3} (x-1)^{3/2} + 4 (x-1)^{1/2} + C$$

$$(3) -\frac{1}{12} (1-2x)^{3/2} + \frac{1}{10} (1-2x)^{5/2} - \frac{1}{28} (1-2x)^{7/2} + C$$

$$(4) \frac{1}{16} \left[ \frac{9}{7} (5+4x)^{7/3} - \frac{69}{4} (5+4x)^{4/3} \right] + C$$

$$(5) \frac{4}{3} (x+3)^{3/2} - 14 (x+3)^{1/2} + C$$

$$(6) \frac{1}{8} \left[ \frac{2}{5} (2x-1)^{5/2} + \frac{4}{3} (2x-1)^{3/2} - 6 (2x-1)^{1/2} \right] + C$$

$$(7) \left[ \frac{3}{10} (x+2)^{10/3} - \frac{12}{7} (x+2)^{7/3} + 3 (x+2)^{4/3} \right] \Big|_{-2}^6 = \frac{4752}{35} \approx 135.77$$

$$(8) \frac{1}{18} \left[ \frac{3}{8} (3x^2-4)^{8/3} + \frac{12}{5} (3x^2-4)^{5/3} \right] + C$$

$$(9) \frac{1}{75} [ \ln |5x^3-7| - 7 (5x^3-7)^{-1} ] + C$$

Determine:

$$(1) \int \frac{2x+1}{x+3} dx ; (2) \int \frac{5x+7}{2x-3} dx ; (3) \int \frac{x^3+6x^2-6x+3}{x^2+6x-7} dx$$

$$(4) \int \frac{15x^3-31x^2+14x+4}{5x-7} dx ; (5) \int \frac{e^{3x}-2e^{2x}+5e^x}{e^x+1} dx$$

$$(6) \int \frac{3x^5-41x^4+60x^3+20x^2-27x}{x^3-12x^2+7} dx ; (7) \int \frac{21x^3-63x^2+17x-3}{7x^2-21x+5} dx$$

Answers:

$$(1) 2x - 5 \ln|x+3| + C ; (2) \frac{5}{2}x + \frac{29}{4} \ln|2x-3| + C$$

$$(3) \frac{x^2}{2} + \frac{1}{2} \ln|x^2+6x-7| + C ; (4) x^3 - x^2 + \frac{4}{5} \ln|5x-7| + C$$

$$(5) \text{ let } u = e^x, \text{ then divide } \rightarrow \frac{1}{2} e^{2x} - 3e^x + 8 \ln(e^x+1) + C$$

$$(6) x^3 - \frac{5}{2}x^2 - \frac{1}{3} \ln|x^3-12x^2+7| + C$$

$$(7) \frac{3}{2}x^2 + \frac{1}{7} \ln|7x^2-21x+5| + C$$

TRIGONOMETRIC INTEGRALS

$$(1) \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx ; (2) \int 5x \tan 3x^2 dx ; (3) \int \sec 10x \tan 10x dx$$

$$(4) \int \frac{\cos (\ln x)}{x} dx ; (5) \int x^2 \sec^2 x^3 dx ; (6) \int \cot(-3x) \csc(-3x) dx$$

$$(7) \int e^x \sec(e^x) dx ; (8) \int \frac{\csc^2 \sqrt[3]{x}}{x^{2/3}} dx = \int \frac{(\csc \sqrt[3]{x})^2}{x^{2/3}} dx$$

$$(9) \int \sin x \sin(\cos x) dx ; (10) \int \sin^5 x \cos x dx$$

$$(11) \int \cos^3 2x \sin 2x dx ; (12) \int x \tan^5 x^2 \sec^2 x^2 dx$$

$$(13) \int \csc^2 4x \cot 4x dx ; (14) \int \frac{\cot^4 5x}{\sin^2 5x} dx$$

$$(15) \int x \sec^6 3x^2 \tan 3x^2 dx ; (16) \int \frac{\sqrt{\cot 2x}}{\sin^2 2x} dx$$

$$(17) \int \csc^5 x \cot x dx ; (18) \int \frac{\sin \sqrt{x} \cos \sqrt{x}}{\sqrt{x}} dx$$

$$(19) \int \frac{\cos \left( \frac{1}{x} \right)}{x^2 \sin \left( \frac{1}{x} \right)} dx$$

Answers:

- (1)  $-2 \cos \sqrt{x} + C$  ; (2)  $-\frac{5}{6} \ln |\cos 3x^2| + C$  ; (3)  $\frac{1}{10} \sec 10x + C$
- (4)  $\sin(\ln x) + C$  ; (5)  $\frac{1}{3} \tan x^3 + C$  ; (6)  $\frac{1}{3} \csc(-3x) + C$
- (7)  $\ln |\sec(e^x) + \tan(e^x)| + C$  ; (8)  $-3 \cot\left(\frac{3}{\sqrt{x}}\right) + C$
- (9)  $\cos(\cos x) + C$  ; (10)  $\frac{1}{6} \sin^6 x + C$
- (11)  $-\frac{1}{8} \cos^4 2x + C$  ; (12)  $\frac{1}{12} \tan^6 x^2 + C$
- (13)  $-\frac{1}{8} \cot^2 4x + C$  or  $-\frac{1}{8} \csc^2 4x + C$  ; (14)  $-\frac{1}{25} \cot^5 5x + C$
- (15)  $\frac{1}{36} \sec^6 3x^2 + C$  ; (16)  $-\frac{1}{3} (\cot 2x)^{3/2} + C$
- (17)  $-\frac{1}{5} \csc^5 x + C$  ; (18)  $\sin^2 \sqrt{x} + C$  or  $-\cos^2 \sqrt{x} + C$
- (19)  $-\ln \left| \sin\left(\frac{1}{x}\right) \right| + C$

TRIGONOMETRIC INTEGRALS

$$(1) \int \sin^3 x \, dx = \frac{1}{3} \cos^3 x - \cos x + C$$

$$(2) \int \sin^4 x \, dx = \frac{3}{8}x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

$$(3) \int \cos^4 x \sin^2 x \, dx = \frac{1}{16}x - \frac{1}{64} \sin 4x + \frac{1}{48} \sin^3 2x + C$$

$$(4) \int \tan^5 x \, dx = \frac{1}{4} \tan^4 x - \sec^2 x + \ln|\sec x| + C$$

$$(5) \int \sqrt{\tan x} \sec^2 x \, dx = \frac{2}{3} \tan^{3/2} x + C$$

$$(6) \int \sqrt{\sec x} \tan^3 x \, dx = \frac{2}{5} \sec^{5/2} x - 2 \sqrt{\sec x} + C$$

$$(7) \int \sec^4 x \tan^3 x \, dx = \frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + C$$

$$(8) \int (\csc x)^{2/3} \cot^3 x \, dx = \frac{3}{2} \csc^{2/3} x - \frac{3}{8} \csc^{8/3} x + C$$

$$(9) \int \cot^3 x \, dx = -\frac{1}{2} \cot^2 x + \ln|\csc x| + C$$

$$(10) \int \sqrt[3]{\cot x} \csc^4 x \, dx = -\frac{3}{4} \cot^{4/3} x - \frac{3}{10} \cot^{10/3} x + C$$

$$(11) \int \frac{\tan(\ln x) \, dx}{x} = \ln |\sec(\ln x)| + C$$

$$(12) \int \sin^3 6x \, dx = \frac{1}{18} \cos^3 6x - \frac{1}{6} \cos 6x + C$$

$$(13) \int x \sqrt{\sec(x^2)} \tan^3(x^2) \, dx = \frac{1}{5} \sec^{5/2}(x^2) - \sqrt{\sec(x^2)} + C$$