## Balancing Chemical Equations

An example:
Consider the following chemical reaction:

$$
\begin{aligned}
& \mathrm{N}_{2} \mathrm{H}_{4} \\
& \text { (Hydrazine) }) \\
& +\underset{2}{+} \text { (Dinitrogen tetroxide) }
\end{aligned}
$$

Let: $\quad x_{1}=$ number of molecules of $N_{2} H_{4}$ in the reaction $x_{2}=$ number of molecules of $\mathrm{N}_{2} \mathrm{O}_{4}$ in the reaction $x_{3}=$ number of molecules of $\mathrm{N}_{2}$ in the reaction $x_{4}=$ number of molecules of $\mathrm{H}_{2} \mathrm{O}$ in the reaction
so that:

$$
x_{1} \mathrm{~N}_{2} \mathrm{H}_{4}+x_{2} \mathrm{~N}_{2} \mathrm{O}_{4} \rightarrow x_{3} \mathrm{~N}_{2}+x_{4} \mathrm{H}_{2} \mathrm{O}
$$

Number of atoms of Nitrogen: $\quad 2 x_{1}+2 x_{2}=2 x_{3}$ or $x_{1}+x_{2}=x_{3}$
Number of atoms of Hydrogen: $\quad 4 x_{1}=2 x_{4}$ or $2 x_{1}=x_{4}$
Number of atoms of Oxygen: $\quad 4 x_{2}=x_{4}$
The resulting system of equations is:

$$
\begin{gathered}
\left\{\begin{array}{ccc}
x_{1}+x_{2}-x_{3} & =0 \\
2 x_{1} & & -x_{4}=0 \\
& 4 x_{2} & -x_{4}=
\end{array}\right\} \\
{\left[\begin{array}{ccccc}
1 & 1 & -1 & 0 & 0 \\
2 & 0 & 0 & -1 & 0 \\
0 & 4 & 0 & -1 & 0
\end{array}\right] \Rightarrow\left[\begin{array}{ccccc}
1 & 0 & 0 & -1 / 2 & 0 \\
0 & 1 & 0 & -1 / 4 & 0 \\
0 & 0 & 1 & -3 / 4 & 0
\end{array}\right] \Rightarrow \begin{array}{l}
x_{1}=\frac{1}{2} t \\
x_{2}=\frac{1}{4} t \\
x_{3}=\frac{3}{4} t \\
x_{4}=t
\end{array}}
\end{gathered}
$$

There are many infinitely many choices for $t$. Any value of $t$ such that $x_{1}, x_{2}, x_{3}, x_{4}$ are natural numbers (i.e. 1, 2, 3, ...) will suffice.

$$
x_{1}=2
$$

For example, $t=4 \quad x_{2}=1$ or example, let $t=4$, so that $\quad x_{3}=3$ resulting in the balanced equation:

$$
\begin{gathered}
x_{4}=4 \\
2 N_{2} H_{4}+N_{2} O_{4} \rightarrow 3 N_{2}+4 H_{2} O
\end{gathered}
$$

## Problems

Balance the following chemical equations. Set up the appropriate system of equations and use an augmented matrix to solve the system. Define all variables in words.

$$
\begin{equation*}
\mathrm{Fe}_{2} \mathrm{O}_{3}+\mathrm{CO} \rightarrow \mathrm{Fe}+\mathrm{CO}_{2} \tag{1}
\end{equation*}
$$ (Hematite) (Carbon monoxide) (Iron) (Carbon dioxide)

$$
\begin{gather*}
\mathrm{C}_{7} \mathrm{H}_{8}  \tag{2}\\
\text { (Toluene) })
\end{gathered} \underset{\substack{\left.\mathrm{HNO}_{3} \\
\text { (Nitric Acid) }\right)}}{\rightarrow} \quad \begin{gathered}
\mathrm{C}_{7} \mathrm{H}_{5} \mathrm{O}_{6} \mathrm{~N}_{3} \\
(\text { TNT })
\end{gather*} \underset{\text { (Water) }}{\mathrm{H}_{2} \mathrm{O}}
$$

$$
\mathrm{MnSO}_{4}+\mathrm{KMnO}_{4}+\mathrm{H}_{2} \mathrm{O} \rightarrow \mathrm{MnO}_{2}+\mathrm{K}_{2} \mathrm{SO}_{4}+\mathrm{H}_{2} \mathrm{SO}_{4}
$$

