Math - Calculus II IMPROPER INTEGRALS

(1) $\int_0^{\infty} e^{-x}\,dx$; (2) $\int_{-\infty}^{1} e^{x}\,dx$; (3) $\int_0^{\infty} x\,e^{-x}\,dx$; (4) $\int_5^{\infty} \frac{dx}{\sqrt{x-1}}$

(5) $\int_{-\infty}^{0} x^2\,e^{x}\,dx$; (6) $\int_{-\infty}^{\infty} e^{-|x|}\,dx$; (7) $\int_{-\infty}^{\infty} x\,e^{-x^2}\,dx$

(8) $\int_e^{\infty} \frac{dx}{x\,(\ln x)^2}$; (9) $\int_{-\infty}^{\infty} \frac{dx}{x^2+16}$; (10) $\int_{-\infty}^{\infty} x^5\,dx$; (11) $\int_1^{\infty} \ln x\,dx$

(12) $\int_0^{\infty} e^{-x}\cos x\,dx$; (13) $\int_0^{\infty} e^{-2x}\,dx$; (14) $\int_{-\infty}^{-1} \frac{1}{x^3}\,dx$

(15) $\int_0^{4} \frac{dx}{(4-x)^{3/2}}$; (16) $\int_0^{4} \frac{dx}{(4-x)^{2/3}}$; (17) $\int_{-2}^{2} \frac{dx}{(x+1)^3}$; (18) $\int_0^{2} \frac{dx}{\sqrt{4-x^2}}$

(19) $\int_{\pi/4}^{\pi/2} \sec x\,dx$; (20) $\int_0^{4} \frac{x\,dx}{\sqrt{16-x^2}}$; (21) $\int_0^{\infty} \frac{dx}{x^3}$; (22) $\int_0^{\pi/2} \tan\theta\,d\theta$

(23) $\int_0^{\pi/2} \frac{dt}{1-\sin t}$; (24) $\int_0^{2} \frac{dx}{(x-1)^{2/3}}$; (25) $\int_0^{4} \frac{dx}{x^2-2x-3}$

(26) $\int_2^{\infty} \frac{dx}{x\sqrt{x^2-4}}$; (27) $\int_0^{\infty} \ln x\,dx$; (28) $\int_{-2}^{0} \frac{dx}{(x+1)^{1/3}}$; (29) $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

(30) $\int_{\frac{1}{2}}^{\infty} \frac{dx}{x\,(\ln x)^{1/5}}$; (31) $\int_0^{2} \frac{x\,dx}{1-x}$

Answers:

(1) 1 ; (2) e ; (3) 1 ; (4) div ; (5) 2 ; (6) 2 ; (7) 0 ; (8) 1 ; (9) $\frac{\pi}{4}$; (10) div

(11) div ; (12) $\frac{1}{2}$; (13) $\frac{1}{2}$; (14) $-\frac{1}{2}$; (15) div ; (16) $3\sqrt[3]{4}$; (17) div

(18) $\frac{\pi}{2}$; (19) div ; (20) 4 ; (21) div ; (22) div ; (23) div ; (24) 6 ; (25) div

(26) $\frac{\pi}{4}$; (27) div ; (28) 0 ; (29) 2 ; (30) div ; (31) div

Detailed Answers:

(1) $\lim_{t\to\infty} \int_0^t e^{-x}\,dx = 1$; (2) $\lim_{t\to\infty} \int_t^1 e^{x}\,dx = e$

(3) $\lim_{t\to\infty} \int_0^t x\,e^{-x}\,dx = 1$ (by parts) ; (4) $\lim_{t\to\infty} \int_5^t \frac{dx}{\sqrt{x-1}} =$ (div)

(5) $\lim_{t\to-\infty} \int_t^0 x^2 e^{x}\,dx = 2$ (by parts)

(6) $\int_{-\infty}^0 e^{x}\,dx + \int_0^\infty e^{-x}\,dx$ (recall: $|x| = \pm x$) $= \lim_{t\to-\infty} \int_t^0 e^{x}\,dx + \lim_{s\to\infty} \int_0^s e^{-x}\,dx = 2$

(7) (subst.) $\lim_{t\to-\infty} \int_t^0 xe^{-x^2}\,dx + \lim_{s\to\infty} \int_0^s xe^{-x^2}\,dx = 0$

(8) (subst.) $\lim_{t\to\infty} \int_e^t \frac{dx}{x\,(\ln x)^2} = 1$

(9) (trig subst.) $\lim_{t\to-\infty} \int_t^0 \frac{dx}{x^2+16} + \lim_{s\to\infty} \int_0^s \frac{dx}{x^2+16} = \frac{\pi}{4}$

(10) $\lim_{t\to-\infty} \int_t^0 x^5\,dx + \lim_{s\to\infty} \int_0^s x^5\,dx =$ div

(11) (parts) $\lim_{t\to\infty} \int_1^t \ln x\,dx =$ div

(12)

(queeze theo) (parts) $\lim_{t\to\infty} \int_0^t e^{-x}\cos x\,dx = \frac{1}{2}$

Hint: $\lim_{x\to\infty} \frac{\cos x}{e^x} = 0$ ➡ Theorem: $-\frac{1}{e^x} \le \frac{\cos x}{e^x} \le \frac{1}{e^x}$

as $x\to\infty$, $-\frac{1}{e^x} \to 0$ and $\frac{1}{e^x} \to 0$ then $\frac{\cos x}{e^x} \to 0$

(13) $\lim_{t\to\infty} \int_0^t e^{-2x}\,dx = \frac{1}{2}$; (14) $\lim_{t\to-\infty} \int_t^{-1} \frac{1}{x^3}\,dx = -\frac{1}{2}$

(15) $\lim_{a\to4^-} \int_0^a \frac{dx}{(4-x)^{3/2}} =$ div ; (16) $\lim_{a\to4^-} \int_0^a \frac{dx}{(4-x)^{2/3}} = 3\sqrt[3]{4}$

(17) $\lim_{a\to -1^-} \int_{-2}^{a} \frac{dx}{(x+1)^3} + \lim_{b\to -1^+} \int_{b}^{2} \frac{dx}{(x+1)^3} = \text{div}$

(18) (trig subs.) $\lim_{a\to 2^-} \int_{0}^{a} \frac{dx}{\sqrt{4-x^2}} = \frac{\pi}{2}$; (19) $\lim_{a\to \frac{\pi}{2}^-} \int_{\pi/4}^{a} \sec x\, dx = \text{div}$

(20) $\lim_{a\to 4^-} \int_{0}^{a} \frac{x\, dx}{\sqrt{16-x^2}} = 4$; (21) $\lim_{a\to 0^+} \int_{a}^{1} \frac{dx}{x^3} + \lim_{t\to \infty} \int_{1}^{t} \frac{dx}{x^3} = \text{div}$

(22) $\lim_{a\to \frac{\pi}{2}^-} \int_{0}^{a} \tan\theta\, d\theta = \text{div}$

(23) multiply by $\frac{1+\sin t}{1+\sin t}$ to integrate: $\lim_{a\to \frac{\pi}{2}^-} \int_{0}^{a} \frac{dt}{1-\sin t} = \text{div}$

(24) $\lim_{a\to 1^-} \int_{0}^{1} \frac{dx}{(x-1)^{2/3}} + \lim_{b\to 1^+} \int_{b}^{2} \frac{dx}{(x-1)^{2/3}} = 6$

(25) (partial fractions) $\lim_{a\to 3^-} \int_{0}^{a} \frac{dx}{(x-3)(x+1)} + \lim_{b\to 3^+} \int_{b}^{4} \frac{dx}{(x-3)(x+1)} = \text{div}$

(26) (trig subs.) $\lim_{a\to 2^+} \int_{a}^{3} \frac{dx}{x\sqrt{x^2-4}} + \lim_{t\to \infty} \int_{3}^{t} \frac{dx}{x\sqrt{x^2-4}} = \frac{\pi}{4}$

(27) (parts) $\lim_{a\to 0^+} \int_{a}^{1} \ln x\, dx + \lim_{t\to \infty} \int_{1}^{t} \ln x\, dx = \text{div}$

(28) $\lim_{a\to -1^-} \int_{-2}^{a} \frac{dx}{(x+1)^{1/3}} + \lim_{b\to -1^+} \int_{b}^{0} \frac{dx}{(x+1)^{1/3}} = 0$

(29) (subst.) $\lim_{a\to 0^+} \int_{a}^{1} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx + \lim_{t\to \infty} \int_{1}^{t} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx = 2$

(30) (subst.) $\lim_{a\to 1^-} \int_{\frac{1}{2}}^{a} \frac{dx}{x(\ln x)^{1/5}} + \lim_{b\to 1^+} \int_{b}^{e} \frac{dx}{x(\ln x)^{1/5}} + \lim_{t\to \infty} \int_{e}^{t} \frac{dx}{x(\ln x)^{1/5}} = \text{div}$

(31) (alg. subst. or long div) $\lim_{a\to 1^-} \int_{0}^{a} \frac{x\, dx}{1-x} + \lim_{b\to 1^+} \int_{b}^{2} \frac{x\, dx}{1-x} = \text{div}$