Linear Programming Problems

1. Maximize $z = 6x_1 + 4x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & & & \leq & 700 \\
& & x_2 & \leq & 900 \\
3x_1 & + & x_2 & \leq & 2400 \\
x_1 & + & 2x_2 & \leq & 1600 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(640,\ 480)$, $z_{max} = 5760$

2. Maximize $z = x_1 + x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & + & x_2 & \geq & 1 \\
5x_1 & + & 10x_2 & \leq & 50 \\
& & x_2 & \leq & 4 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(10,\ 0)$, $z_{max} = 10$

3. Maximize $z = 3x_1 + 2x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & + & x_2 & \geq & 1 \\
-5x_1 & + & x_2 & \leq & 0 \\
-x_1 & + & 5x_2 & \geq & 0 \\
x_1 & + & x_2 & \leq & 6 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(5,\ 1)$, $z_{max} = 17$

4. Maximize $z = 4x_1 + 3x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & + & x_2 & \leq & 10 \\
2x_1 & + & 2x_2 & \geq & 40 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: Infeasible.

5. Maximize $z = -3x_1 + 4x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & - & x_2 & \geq & 0 \\
2x_1 & + & 2x_2 & \leq & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(2,\ 2)$, $z_{max} = 2$

6. Maximize $z = 2x_1 + x_2$ subject to

$$
\begin{array}{rrrcr}
11x_1 & + & 3x_2 & \geq & 33 \\
8x_1 & + & 5x_2 & \leq & 40 \\
7x_1 & + & 10x_2 & \leq & 70 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(5,\ 0)$, $z_{max} = 10$

7. Minimize $z = 5x_1 + 8x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & & & \leq & 400 \\
& & x_2 & \geq & 200 \\
x_1 & + & x_2 & = & 500 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(300,\ 200)$, $z_{min} = 3100$

8. Minimize $z = 22x_1 + 6x_2$ subject to

$$
\begin{array}{rrrcr}
11x_1 & + & 3x_2 & \geq & 33 \\
8x_1 & + & 5x_2 & \leq & 40 \\
7x_1 & + & 10x_2 & \leq & 70 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(3,\ 0)$ or $\left(\frac{45}{31}, \frac{176}{31}\right)$, $z_{min} = 66$

9. Minimize $z = -2x_1 + 6x_2$ subject to

$$
\begin{array}{rrrcr}
3x_1 & + & 2x_2 & \leq & 6 \\
x_1 & - & x_2 & \geq & -1 \\
2x_1 & - & x_2 & \geq & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(2,\ 0)$ $z_{min} = -4$

10. Maximize $z = 8x_1 + 5x_2$ subject to

$$
\begin{array}{rrrcr}
10x_1 & + & 9x_2 & \leq & 90 \\
4x_1 & + & 3x_2 & \geq & 12 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(9,\ 0)$ $z_{max} = 72$

11. Maximize $z = 6x_1 + 2x_2$ subject to

$$
\begin{array}{rrrcr}
5x_1 & + & 6x_2 & \geq & 30 \\
7x_1 & + & 8x_2 & \leq & 56 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(8,\ 0)$ $z_{max} = 48$

12. Maximize $z = x_1 - 2x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & - & 2x_2 & \leq & 2 \\
x_1 & + & x_2 & \geq & 4 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{10}{3}, \frac{2}{3}\right)$, $z_{max} = 2$

13. Maximize $z = 3x_1 + 2x_2$ subject to

$$
\begin{array}{rrrcr}
x_1 & + & x_2 & \leq & 10 \\
3x_1 & + & x_2 & \geq & 3 \\
x_1 & - & x_2 & \leq & 6 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(8,\ 2)$, $z_{max} = 28$

14. Minimize $z = 3x_1 + 4x_2$ subject to

$$
\begin{array}{rcrcl}
3x_1 & + & x_2 & \leq & 9 \\
x_1 & + & x_2 & \geq & 7 \\
x_1 & + & 2x_2 & \geq & 8 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(1, 6)$ $z_{min} = 27$

15. Minimize $z = 30x_1 + 40x_2$ subject to

$$
\begin{array}{rcrcl}
2x_1 & - & x_2 & \geq & 12 \\
x_1 & + & x_2 & \geq & 9 \\
x_1 & + & 3x_2 & \geq & 15 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{51}{7}, \frac{18}{7}\right)$, $z_{min} = \frac{2250}{7}$

16. Maximize $z = 2x_1 + 8x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & + & x_2 & \geq & 9 \\
3x_1 & + & x_2 & \geq & 12 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: unbounded case

17. Minimize $z = 8x_1 + 9x_2 + 5x_3$ subject to

$$
\begin{array}{rcrcrcl}
2x_1 & + & 3x_2 & + & x_3 & \geq & 2 \\
5x_1 & + & 2x_2 & - & 3x_3 & \geq & 4 \\
7x_1 & + & 6x_2 & + & 4x_3 & \geq & 5 \\
x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{8}{11}, \frac{2}{11}, 0\right)$, $z_{min} = \frac{82}{11}$

18. Minimize $z = 20x_1 + 20x_2 + 40x_3$ subject to

$$
\begin{array}{rcrcrcl}
2x_1 & + & 5x_2 & + & 6x_3 & \geq & 6 \\
6x_1 & + & 2x_2 & + & 12x_3 & \geq & 9 \\
5x_1 & + & 4x_2 & + & 8x_3 & \geq & 8 \\
x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{3}{8}, \frac{21}{40}, \frac{29}{80}\right)$, $z_{min} = 37$

19. Minimize $z = 65x_1 + 120x_2 + 70x_3$ subject to

$$
\begin{array}{rcrcrcl}
x_1 & + & x_2 & + & 2x_3 & \geq & 5 \\
x_1 & + & 2x_2 & + & 2x_3 & \geq & 6 \\
2x_1 & + & 4x_2 & + & x_3 & \geq & 8 \\
x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{4}{3}, 1, \frac{4}{3}\right)$, $z_{min} = 300$

20. Maximize $z = 3x_1 + 4x_2$ subject to

$$
\begin{array}{rcrcl}
2x_1 & + & 3x_2 & = & 4 \\
2x_1 & + & x_2 & \leq & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $\left(\frac{1}{2}, 1\right)$ $z_{max} = 5\frac{1}{2}$

21. Maximize $z = 2x_1 + 4x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & + & 2x_2 & \leq & 3 \\
x_1 & + & x_2 & = & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(1, 1)$ $z_{max} = 6$

22. Maximize $z = 2x_1 - x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & + & 3x_2 & \leq & 6 \\
x_1 & - & x_2 & = & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(3, 1)$ $z_{max} = 5$

23. Maximize $z = x_1 + 2x_2$ subject to

$$
\begin{array}{rcrcl}
2x_1 & + & x_2 & \leq & 4 \\
x_1 & + & x_2 & = & 3 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(0, 3)$ $z_{max} = 6$

24. Minimize $z = 12x_1 + 16x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & + & x_2 & = & 50 \\
x_1 & & & \leq & 30 \\
& & x_2 & \geq & 20 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(30, 20)$ $z_{min} = 680$

25. Minimize $z = 6x_1 + 10x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & & & \leq & 12 \\
& & x_2 & = & 18 \\
3x_1 & + & 2x_2 & \geq & 54 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(6, 18)$ $z_{min} = 216$

26. Maximize $z = x_1 + x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 & + & x_2 & \leq & 2 \\
2x_1 & + & x_2 & \leq & 3 \\
2x_1 & + & x_2 & = & 2 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(0, 2)$ $z_{max} = 2$

27. Maximize $z = 4x_1 + 12x_2$ subject to

$$
\begin{array}{rcrcl}
3x_1 & + & x_2 & \leq & 180 \\
x_1 & + & 2x_2 & = & 100 \\
-2x_1 & + & 2x_2 & \leq & 40 \\
x_1 & , & x_2 & \geq & 0
\end{array}
$$

Solution: $(20, 40)$ $z_{max} = 560$

28. Minimize $z = x_1 + x_2$ subject to

$$
\begin{array}{rcrcl}
x_1 &+& x_2 &\leq& 2 \\
2x_1 &+& x_2 &\leq& 3 \\
2x_1 &+& x_2 &=& 2 \\
x_1 &,& x_2 &\geq& 0
\end{array}
$$

Solution: $(1, 0)$ $z_{min} = 1$

29. Maximize $z = 5x_1 + 15x_2$ subject to

$$
\begin{array}{rcrcl}
4x_1 &+& x_2 &\leq& 200 \\
x_1 &+& 3x_2 &\geq& 120 \\
-x_1 &+& 3x_2 &=& 150 \\
x_1 &,& x_2 &\geq& 0
\end{array}
$$

Solution: $\left(\frac{450}{13}, \frac{800}{13}\right)$ $z_{max} = \frac{14250}{13}$

30. Maximize $z = 6x_1 + 4x_2$ subject to

$$
\begin{array}{rcrcl}
3x_1 &+& 2x_2 &\leq& 60 \\
2x_1 &+& 3x_2 &\geq& 24 \\
x_1 &+& x_2 &=& 25 \\
x_1 &,& x_2 &\geq& 0
\end{array}
$$

Solution: $(10, 15)$ $z_{max} = 120$

31. Minimize $z = x_1 + 4x_2 + x_3$ subject to

$$
\begin{array}{rcrcrcl}
-x_1 &+& x_2 &+& x_3 &=& 2 \\
-x_1 &+& 2x_2 & & &\geq& 1 \\
x_1 &,& x_2 &,& x_3 &\geq& 0
\end{array}
$$

Solution: $\left(0, \frac{1}{2}, \frac{3}{2}\right)$, $z_{min} = 3.5$

32. Minimize $z = 4x_1 + 3x_2 + x_3$ subject to

$$
\begin{array}{rcrcrcl}
x_1 &+& x_2 &+& x_3 &=& 9 \\
4x_1 &-& 5x_2 &-& 5x_3 &\geq& 0 \\
-x_1 & & &+& x_3 &\leq& 0 \\
x_1 &,& x_2 &,& x_3 &\geq& 0
\end{array}
$$

Solution: $(5, 0, 4)$, $z_{min} = 24$

33. Maximize $z = 7x_1 + 2x_2 + 5x_3$ subject to

$$
\begin{array}{rcrcrcl}
2x_1 &+& x_2 &+& x_3 &\leq& 50 \\
x_1 &+& x_2 &+& 2x_3 &\leq& 40 \\
x_1 &+& 3x_2 &+& x_3 &=& 35 \\
x_1 &,& x_2 &,& x_3 &\geq& 0
\end{array}
$$

Solution: $\left(\frac{135}{7}, \frac{15}{7}, \frac{65}{7}\right)$, $z_{max} = \frac{1300}{7}$

34. Maximize $z = 2x_1 + 3x_2$ subject to

$$
\begin{array}{rcrcl}
2x_1 &+& x_2 &=& 2 \\
-x_1 &+& x_2 &=& 1 \\
x_1 &,& x_2 &\geq& 0
\end{array}
$$

Solution: $\left(\frac{1}{3}, \frac{4}{3}\right)$, $z_{max} = \frac{14}{3}$

35. Minimize $z = 3x_1 + 2x_2 + 3x_3 + x_4$ subject to

$$
\begin{array}{rcrcrcrcl}
8x_1 &+& 2x_2 &+& 2x_3 &-& 4x_4 &=& 4 \\
3x_1 &-& 3x_2 &+& x_3 &-& x_4 &=& 18 \\
x_1 &,& x_2 &,& x_3 &,& x_4 &\geq& 0
\end{array}
$$

Solution: $(17, 0, 0, 33)$, $z_{min} = 54$

36. Minimize $z = 4x_1 + 2x_2 + x_3 - 5x_4$ subject to

$$
\begin{array}{rcrcrcrcl}
x_1 &-& x_2 &+& 2x_3 &-& 5x_4 &=& 13 \\
3x_1 &+& 9x_2 &-& 6x_3 &-& 3x_4 &=& 3 \\
x_1 &,& x_2 &,& x_3 &,& x_4 &\geq& 0
\end{array}
$$

Solution: $(0, 7, 10, 0)$, $z_{min} = 24$

37. Minimize $z = -x_1 - 3x_2 - 14x_3 - 4x_4$ subject to

$$
\begin{array}{rcrcrcrcl}
x_1 &-& 2x_2 &+& 2x_3 &-& 2x_4 &=& 4 \\
4x_1 &-& 6x_2 &+& 9x_3 &-& 3x_4 &=& 17 \\
x_1 &,& x_2 &,& x_3 &,& x_4 &\geq& 0
\end{array}
$$

Solution: $(2, 0, 1, 0)$, $z_{min} = -16$

38. Minimize $z = -2x_1 + 6x_2$ subject to

$$
\begin{array}{rcrcr}
3x_1 &+& 2x_2 &\leq& 6 \\
x_1 &-& x_2 &\geq& -1 \\
2x_1 &-& x_2 &\geq& 2 \\
x_1 &,& x_2 &\geq& 0
\end{array}
$$

Solution: $(2, 0)$, $z_{min} = -4$

39. Minimize $z = 2x_1 + 3x_2 + 4x_3$ subject to

$$
\begin{array}{rcrcrcl}
2x_1 &+& x_2 &+& 3x_3 &\geq& 12 \\
x_1 &+& 2x_2 &+& x_3 &\geq& 18 \\
x_1 &,& x_2 &,& x_3 &\geq& 0
\end{array}
$$

Solution: $(2, 8, 0)$, $z_{min} = 28$