

1. Simplify: (the result should have no trig)

(a) $\sin \left(\arctan \left(\frac{2x+1}{3} \right) \right)$; (b) $\sec \left(\arcsin \left(\frac{2}{x} \right) \right)$

(c) $\operatorname{arcsec}(-\sqrt{2})$

2. Differentiate:

(a) $y = e^{\arctan(x+\sqrt{x})} - \arctan(e^{x+\sqrt{x}})$

(b) $y = \sin(x - \arccos x^2)$; (c) $y = \frac{\operatorname{arcsec} 2x}{x}$

3. Evaluate: (exact answers)

(a) $\int_{\frac{1}{4}}^1 x^{-1/2} \arcsin \sqrt{x} \, dx$; (b) $\int_0^1 \frac{3x-1}{x^2+1} \, dx$

(c) $\int_0^{\sqrt{3}} \arctan x \, dx$; (d) $\int_1^e x^2 (\ln x)^2 \, dx$

(e) $\int \cos(\ln x) \, dx$; (f) $\int \frac{\cos(\ln x)}{x} \, dx$; (g) $\int \sqrt{x} e^{\sqrt{x}} \, dx$

(h) $\int \csc^3 x \, dx$; (i) $\int x^3 e^{-5x^4+1} \, dx$

(j) $\int \frac{2\sqrt{x-1}}{\sqrt{x-1}} \, dx$; (k) $\int \frac{\cos 3x}{1 + \sin 3x} \, dx$; (l) $\int (\tan 4x + 3x \csc x^2) \, dx$

(m) $\int \left(\frac{x}{1-2x^2} + \sqrt{1-4x} \right) \, dx$; (n) $\int_1^e x^3 \ln x \, dx$; (o) $\int_0^\pi e^{2x} \sin x \, dx$

(p) $\int \frac{3x^3+2x^2-x+4}{x+1} \, dx$; (q) $\int x \sqrt[3]{2x+1} \, dx$

(r) $\int x^2 \cos 5x \, dx$; (s) $\int \cos^4 3x \, dx$; (t) $\int \cot^5 2x \csc^3 2x \, dx$

(u) $\int_0^{\sqrt{\pi}} x^5 \cos(x^2) \, dx$

Answers:

$$(1 \text{ a}) \quad \frac{2x+1}{\sqrt{9+(2x+1)^2}} \quad ; \quad (1 \text{ b}) \quad \frac{x}{\sqrt{x^2-4}} \quad ; \quad (1 \text{ c}) \quad \frac{5\pi}{4}$$

$$(2 \text{ a}) \quad y' = \frac{e^{\arctan(x+\sqrt{x})}}{1+(x+\sqrt{x})^2} \left(1 + \frac{1}{2\sqrt{x}} \right) - \frac{e^{x+\sqrt{x}}}{1+(e^{x+\sqrt{x}})^2} \left(1 + \frac{1}{2\sqrt{x}} \right)$$

$$(2 \text{ b}) \quad y' = \cos(x - \arccos x^2) \left(1 + \frac{2x}{\sqrt{1-x^4}} \right)$$

$$(2 \text{ c}) \quad y' = \frac{1}{x^2} \left(\frac{1}{\sqrt{4x^2-1}} - \operatorname{arcsec} 2x \right)$$

$$(3 \text{ a}) \quad \frac{5\pi}{6} - \sqrt{3} \quad ; \quad (3 \text{ b}) \quad \frac{3}{2} \ln(2) - \frac{\pi}{4} \quad ; \quad (3 \text{ c}) \quad \sqrt{3} \frac{\pi}{3} - \frac{1}{2} \ln(4)$$

$$(3 \text{ d}) \quad \frac{5}{27}e^3 - \frac{2}{27} \quad ; \quad (3 \text{ e}) \quad \frac{1}{2}x \cos(\ln x) + \frac{1}{2}x \sin(\ln x) + C$$

$$(3 \text{ f}) \quad \sin(\ln x) + C \quad ; \quad (3 \text{ g}) \quad 2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4 e^{\sqrt{x}} + C$$

$$(3 \text{ h}) \quad \frac{1}{2} [-\csc x \cot x + \ln |\csc x - \cot x|] + C \quad ; \quad (3 \text{ i}) \quad -\frac{1}{20} e^{-5x^4+1} + C$$

$$(3 \text{ j}) \quad 2 \frac{2^{\sqrt{x-1}}}{\ln(2)} + C \quad ; \quad (3 \text{ k}) \quad \frac{1}{3} \ln |1 + \sin 3x| + C$$

$$(3 \text{ l}) \quad \frac{1}{4} \ln|\sec 4x| + \frac{3}{2} \ln|\csc x^2 - \cot x^2| + C$$

$$(3 \text{ m}) \quad -\frac{1}{4} \ln|1-2x^2| - \frac{1}{6} (1-4x)^{3/2} + C \quad ; \quad (3 \text{ n}) \quad \frac{3}{16}e^4 + \frac{1}{16}$$

$$(3 \text{ o}) \quad \frac{1}{5}e^{2\pi} + \frac{1}{5}$$

$$(3 \text{ p}) \quad x^3 - \frac{x^2}{2} + 4 \ln|x+1| + C \quad ; \quad (3 \text{ q}) \quad \frac{3}{28}(2x+1)^{7/3} - \frac{3}{16}(2x+1)^{4/3} + C$$

$$(3 \text{ r}) \quad \frac{1}{5}x^2 \sin 5x + \frac{2}{25}x \cos 5x - \frac{2}{125} \sin 5x + C$$

$$(3 \text{ s}) \quad \frac{3}{8}x + \frac{1}{12} \sin 6x + \frac{1}{96} \sin 12x + C$$

$$(3 \text{ t}) \quad -\frac{1}{2} \left[\frac{1}{7} (\csc 2x)^7 - \frac{2}{5} (\csc 2x)^5 + \frac{1}{3} (\csc 2x)^3 \right] + C$$

$$(3 \text{ u}) \quad \left[\frac{1}{2}x^4 \sin(x^2) + x^2 \cos(x^2) - \sin(x^2) \right]_0^{\sqrt{\pi}} = -\pi$$