Review Problems # 4

(1) Find the intersection (if any) of the planes.

(a) $\begin{cases} 3x+6y+4z=18 \\ 3x+3y+4z=15 \\ 3x+2y+4z=14 \end{cases}$ (b) $\begin{cases} 4x+12y+3z=24 \\ 2x+5y+10z=20 \\ 2x+y+2z=8 \end{cases}$

(2) Sketch each of the following pairs of planes

(a) $\begin{cases} x+2y+z=4 \\ x+2y+z=8 \end{cases}$ (b) $\begin{cases} x+2y+z=4 \\ 2x+y+z=5 \end{cases}$

Indicate the intersection, if any. (c) find the line of intersection for (b).

(3) Sketch each of the following planes –
find the intercepts, equations for the traces and state the normal.

(a) $y=3$ (b) $2y+4z=8$

(4) Given the points $A(2,1,-3)$, $B(0,1,0)$, $C(1,-1,2)$

(a) find the equation of the line through A and B

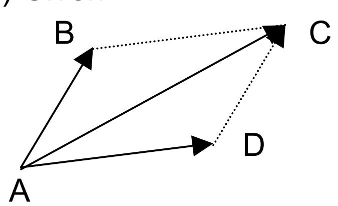
(b) find the equation of the line through A and parallel to $\overrightarrow{BC}$

(c) find the equation of the plane containing the $\triangle ABC$

(d) find the equation of the plane formed by the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ (where $O=(0,0,0)$)

(e) sketch the line in part (a). (f) find $\left\|\overrightarrow{AC}\right\|$

(5) Given



where $\overrightarrow{AB}=\vec{w}$ and $\overrightarrow{AD}=\vec{x}$
find in terms of (a) $\vec{w}$ and $\vec{x}$

(a) $\overrightarrow{AC}$ (b) $\overrightarrow{DA}$ (c) $\overrightarrow{BD}$ (d) $\overrightarrow{DB}$

(6) Why are the following parallel?

(a) (a) $(2,3,-5)$ and $(4,6,-10)$

(b)

$$l_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad l_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

(c) $\begin{cases} 2x-y+z=4 \\ 4x-2y+2z=7 \end{cases}$

(7) Ex 4.3/#33

Review Problems # 4

Solutions:

(1 a) $\left|\begin{array}{ccc|c|c} 4 & 12 & 3 & 24 & 43 \\ 2 & 5 & 10 & 20 & 37 \\ 2 & 1 & 2 & 8 & 13 \end{array}\right| \Rightarrow (x, y, z) = \left(\frac{5}{2}, \frac{19}{21}, \frac{22}{21}\right)$, a single point of intersection

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(1 b) $\left|\begin{array}{ccc|c|c} 3 & 6 & 4 & 18 & 31 \\ 3 & 3 & 10 & 15 & 25 \\ 2 & 1 & 2 & 14 & 23 \end{array}\right| \Rightarrow (x, y, z) = \left(\frac{-4t+12}{3}, 1, t\right)$, a line of intersection

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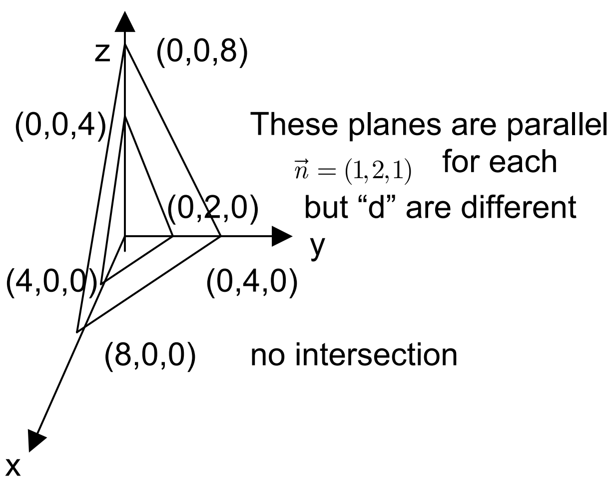
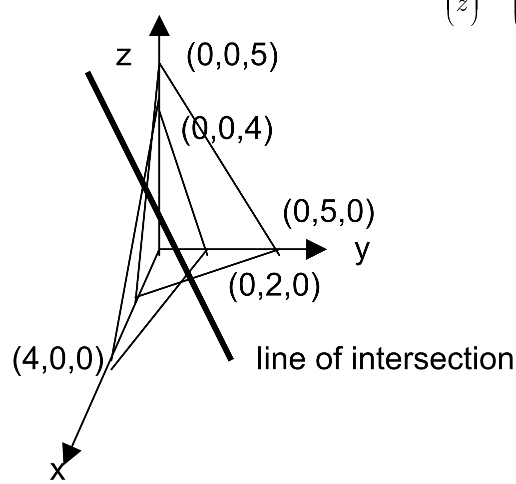
(2 a)

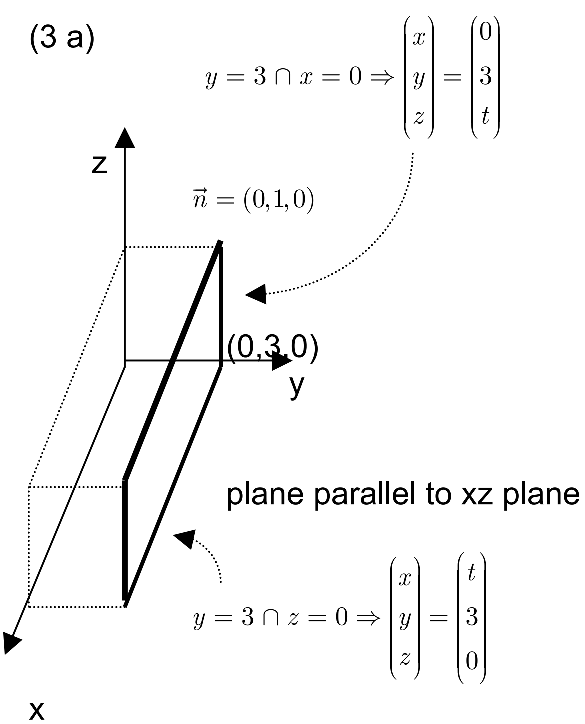


(2 b)

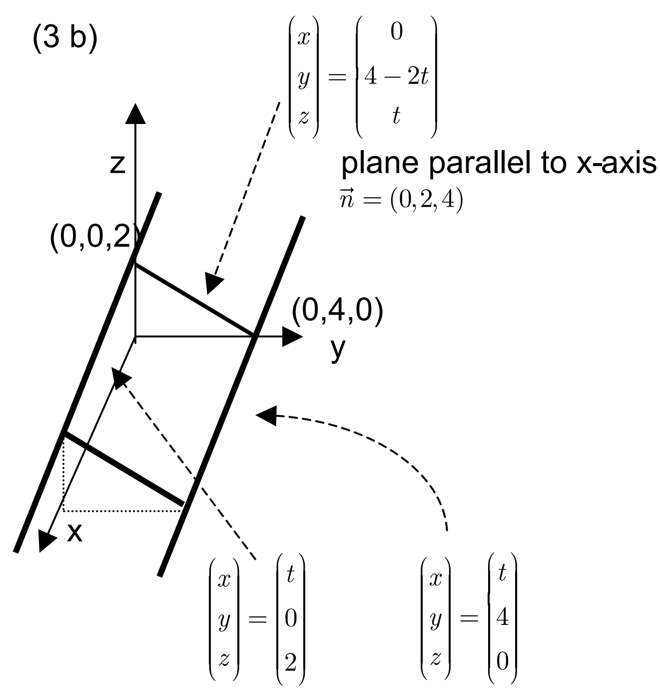


(2 c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$

(3 a)



(3 b)



(4) (a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}}_{\text{or use } B} + t\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$ (b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ (c) $\left[\begin{array}{cccc|c} A & B & C & D & \\ 2 & 1 & -3 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & -1 & 2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} A & B & C & D=t & \\ 1 & 0 & 0 & -\frac{6}{7} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{4}{7} & 0 \end{array}\right] \Rightarrow A = \frac{6}{7}t\ ,\ B = t\ ,\ C = \frac{4}{7}t\ ,\ D = t$

Let $t = 7 \Rightarrow 6x + 7y + 4z = 7$

(d) $\overrightarrow{OA} = (2,1,-3)$; $\overrightarrow{OB} = (0,1,0)$ This plane is a subspace.

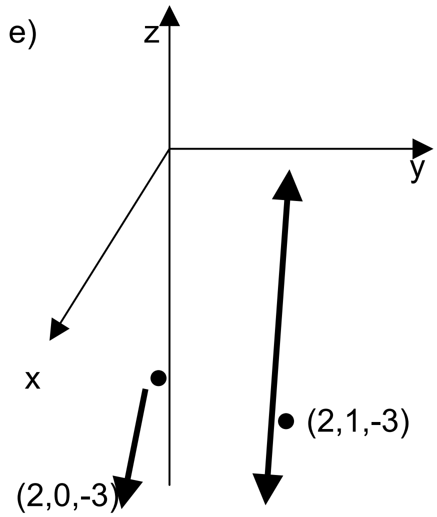
Method 1: $\left[\begin{array}{cc|c} 2 & 0 & x \\ 1 & 1 & y \\ -3 & 0 & z \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 0 & x \\ 0 & 2 & 2y - x \\ 0 & 0 & 2z + 3x \end{array}\right]$; Method 2: (use A, B, C, D method) $\left[\begin{array}{cccc|c} A & B & C & D & \\ 2 & 1 & -3 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} A & B & C & D & \\ \boxed{1} & 0 & -\frac{3}{2} & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \end{array}\right] \Rightarrow \begin{array}{l} A = \frac{3}{2}t \\ B = 0 \\ C = t \\ D = 0 \end{array}$

plane equation: $3x + 2z = 0$

Let $t = 2 \Rightarrow 3x + 0y + 2z = 0$

planes containing the origin can be found this way!
but do <u>not</u> use this method for planes which do
not contain the origin.

(e)



(f) $\overrightarrow{AC} = C - A = (-1,-2,5) \Rightarrow \left\|\overrightarrow{AC}\right\| = \sqrt{1 + 4 + 25} = \sqrt{30}$

(5) (a) $\vec{w} + \vec{x}$ (b) $-\vec{x}$

(c) $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD} \Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \vec{x} - \vec{w}$

(d) $\overrightarrow{DB} = \vec{w} - \vec{x}$

(6) (a) $(4,6,-10) = 2(2,3,-5)$

(b) $\vec{d_1} = \vec{d_2} = (1,2,-1)$; $(2,3,4)$ is a point on l_1 but not on l_2
since $1 + t = 2 \Rightarrow t = 1\ ;\ 2 + 2t = 3 \Rightarrow t = \frac{1}{2}\ ;\ 3 - t = 4 \Rightarrow t = -1$ contradictory t's.

(c) $\begin{array}{ccc} x & y & z \end{array}$

$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 4 \\ 4 & -2 & 2 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -1 & 1 & 4 \\ 0 & 0 & 0 & \underbrace{\ }_{\text{non-zero}} \end{array}\right]$ <u>or</u> the equation can be reduced to

$\begin{cases} 2x - y + z = 4 \\ 2x - y + z = \frac{7}{2} \end{cases}$ the normals are the same but the "D's" are not! $\Rightarrow P_1 // P_2$

(7) see answer in back of textbook