Math - Calculus II REVIEW # 9

Do the following series converge or diverge? State the test used and show all the conditions for applying the test.

(a) $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}-\frac{4}{2^n}\right)$; (b) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{n^3}\right)$; (c) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2n+1}{3n+2}\right)$; (d) $\sum_{n=1}^{\infty} n\,e^{-n^2}$

(e) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$; (f) $\sum_{n=1}^{\infty}\frac{e^{-\sqrt[3]{n}}}{\sqrt[3]{n^2}}$; (g) $\sum_{n=1}^{\infty}\left(\frac{3^n-5^{n+1}}{7^{n+2}}\right)$; (h) $\sum_{n=1}^{\infty}\frac{1}{(3n+4)^2}$

(i) $\sum_{n=1}^{\infty}\frac{\arctan n}{n^2+1}$; (j) $\sum_{n=1}^{\infty}\arctan n$; (k) $\sum_{n=2}^{\infty}\frac{\ln n}{n}$

Answers:

(a) $\sum \frac{1}{n^2}$ (conv p−series) ; $\sum \frac{4}{2^n}$ (conv) G.S. $r=\frac{1}{2}$; sum of given series converges

(b) $\sum \frac{1}{\sqrt{n}}$ (div p−series) ; $\sum \frac{1}{n^3}$ (conv p−series) ; so the given series diverges

(c) $\lim_{n\to\infty}(-1)^n\left(\frac{2n+1}{3n+2}\right)=\pm\frac{2}{3}\neq 0$ (nTT) ; given series diverges

(d) IT: $f(x)=x\,e^{-x^2}\geq 0$, cont , $f'(x)=e^{-x^2}(1-2x^2)<0$ for $x\geq 1$

$\int_1^{\infty} x\,e^{-x^2}\,dx=\lim_{t\to\infty}\left[-\frac{1}{2}e^{-t^2}+\frac{1}{2}e^{-1}\right]=\frac{1}{2}e^{-1}$ convergent

(e) IT: $f(x)=\frac{1}{x(\ln x)^2}\geq 0$, cont , $f'(x)=-\frac{2+\ln x}{x^2(\ln x)^3}<0$ for $x\geq 2$

$\int_2^{\infty}\frac{1}{x(\ln x)^2}\,dx=\lim_{t\to\infty}\left[-\frac{1}{\ln t}+\frac{1}{\ln 2}\right]=\frac{1}{\ln 2}$ convergent

(f) IT: $f(x)=\frac{1}{e^{x^{1/3}}x^{2/3}}\geq 0$, cont , $f'(x)=-\frac{2+x^{1/3}}{3x^{5/3}e^{x^{1/3}}}<0$ for $x\geq 1$

$\int_1^{\infty} e^{-x^{1/3}}x^{-2/3}\,dx=\lim_{t\to\infty}\left[-\frac{3}{e^{t^{1/3}}}+\frac{3}{e}\right]=\frac{3}{e}$ convergent

(g) $\sum \frac{3^n}{7^{n+2}}$ (conv) G.S. $r = \frac{3}{7}$; $\sum \frac{5^{n+1}}{7^{n+2}}$ (conv) G.S. $r = \frac{5}{7}$

diff of 2 conv G.S. is also convergent

(h) converges ; compare with $\sum \frac{1}{n^2}$ (conv p-series) ; ➜ $\frac{1}{(3n+2)^2} \le \frac{1}{n^2}$ for $n \ge 1$

$\sum \frac{1}{n^2}$ converges ➜ $\sum \frac{1}{(3n+2)^2}$ also converges (DC)

(i) converges ; compare with $\sum \frac{1}{n^2}$ (conv p-series) ; ➜ $\frac{\arctan n}{n^2+1} \le \frac{\pi/2}{n^2}$ for $n \ge 1$

$\frac{\pi}{2} \sum \frac{1}{n^2}$ converges ➜ $\sum \frac{\arctan n}{n^2+1}$ also converges (DC)

(j) div since $\lim_{n\to\infty} \arctan n = \frac{\pi}{2} \ne 0$ (nTT)

(k) divergent ; compare with $\sum \frac{1}{n}$ (div p-series) ; ➜ $\frac{1}{n} \le \frac{\ln n}{n}$ for $n \ge 3$

$\sum \frac{1}{n}$ diverges ➜ $\sum \frac{\ln n}{n}$ also diverges (DC)