201-SH3-AB - Exercises #14: Differential Equations

Show that the function y is a particular solution of the given differential equation.

(1) $y = e^{x/2}, \quad 2y'' + 5y' - 3y = 0$

(2) $y = x^3 - \frac{5}{2}x, \quad x\frac{dy}{dx} - 3y = 5x$

(3) $y = 4 + 8xe^x - 3e^x, \quad y'' - 2y' + y - 4 = 0$

(4) $y = x^2(5 + 3\ln(x)), \quad xy' - 2y = 3x^2$

(5) $y = 2e^{\frac{x^4}{4}}, \quad \frac{dy}{dx} = x^3 y$

Solve the following initial value problems.

(6) $y' = y\sin(x), \quad y(0) = 1$

(7) $y' = y^2\cos(x), \quad y(0) = 1$

(8) $\frac{dy}{dx} = 3x^2 y, \quad y(0) = 4$

(9) $y' = y^2(2x + 1), \quad y(-1) = \frac{1}{5}$

(10) $\frac{dy}{dx} = e^{x+2} \cdot y^2, \quad y(-2) = -\frac{1}{2}$

(11) $y' = 6x^2(y - 2), \quad y(2) = 3$

(12) $y' = 3e^{x-y}, \quad y(0) = 2$

(13) $y' = 4xy, \quad y(2) = 1, \quad y > 0$

(14) $y' = \frac{2x^2}{y}, \quad y(1) = 2, \quad y > 0$

(15) $y' = \frac{3x^2}{\sqrt{y}}, \quad y(1) = 9$

(16) $xy' = \frac{4x^2}{y}, \quad y(1) = 2, \quad y > 0$

(17) $y' = \frac{y}{\sqrt{x}}, \quad y(4) = 1, \quad y > 0$

(18) $y' = 2\sqrt{y}e^{3x}, \quad y(0) = \frac{4}{9}$

(19) $y' = 3xy - 2x, \quad y(0) = 1, \quad y > 0$

(20) $y' = 2xy + 3x^2 y, \quad y(2) = 1, \quad y > 0$

(21) $y' = 2x^2 y + 2x^2, \quad y(0) = 0, \quad y > -1$

(22) $y' = \frac{xy}{x^2 + 1}, \quad y(0) = 3, \quad y > 0$

ANSWERS:

(6) $y = e^{1 - \cos(x)}$

(7) $y = \dfrac{1}{1 - \sin(x)}$

(8) $y = 4e^{x^3}$

(9) $y = \dfrac{-1}{x^2 + x - 5}$

(10) $y = \dfrac{-1}{e^{x+2} + 1}$

(11) $y = e^{2x^3 - 16} + 2$

(12) $y = \ln\left(3e^x + e^2 - 3\right)$

(13) $y = e^{2x^2 - 8}$

(14) $y = \sqrt{\frac{4}{3}x^3 + \frac{8}{3}}$

(15) $y = \left(\frac{3}{2}x^3 + \frac{51}{2}\right)^{2/3}$

(16) $y = 2|x|$

(17) $y = e^{2\sqrt{x} - 4}$

(18) $y = \left(\frac{1}{3}e^{3x} + \frac{1}{3}\right)^2$

(19) $y = \frac{2}{3} + \frac{1}{3}e^{3x^2/2}$

(20) $y = e^{x^3 + x^2 - 12}$

(21) $y = e^{(2/3)x^3} - 1$

(22) $y = 3\sqrt{x^2 + 1}$